DERIVATION OF O(3) ELECTRODYNAMICS FROM THE IRREDUCIBLE REPRESENTATIONS OF THE EINSTEIN GROUP

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By considering the irreducible representations of the Einstein group (the Lie group of general relativity), Sachs [1] has shown that the electromagnetic field tensor can be developed in terms of a metric q^μ, which is a set of four quaternion-valued components of four-vector. Using this method, it is shown that the electromagnetic field vanishes [1] in flat spacetime, and that electromagnetism in general is a non-Abelian field theory. In this paper the non-Abelian component of the field tensor is developed to show the presence of the $\mathbf{B}^{(3)}$ field of the O(3) electrodynamics, and the basic structure of O(3) electrodynamics is shown to be a sub-structure of general relativity as developed by Sachs. The extensive empirical evidence for both theories is summarized.

Key words: Irreducible representations of the Einstein group, $\mathbf{B}^{(3)}$ field, O(3) electrodynamics.

1. INTRODUCTION

In a development of Einstein's general relativity using irreducible representations of the Einstein group, Sachs [1] has shown that the electromagnetic field tensor can exist only in curved spacetime and vanishes in the fast spacetime of special relativity. Using this theory [1], Sachs has shown that the structure of electromagnetism is in general non-Abelian. The non-Abelian component of the field tensor is defined through a metric q^μ, which is a set of four quaternion-valued components of a four-vector, components which can be represented by a 2×2 matrix. In condensed notation,

$$q^\mu = (q^{\mu 0}, q^{\mu 1}, q^{\mu 2}, q^{\mu 3}), \tag{1}$$

and the total number of components of q^μ is sixteen. The covariant and second covariant derivatives of q^μ vanish [1], and the line element is given by

$$ds = q^\mu(x) dx_\mu, \tag{2}$$

which in special relativity (flat spacetime) reduces to

$$ds = \sigma^\nu dx_\mu, \tag{3}$$

where σ^μ is a four-vector made up of Pauli matrices:

$$\sigma^\mu = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right). \tag{4}$$

In the limit of special relativity

$$q^\mu q^{\nu *} = q^\nu q^{\mu *} \rightarrow \sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu, \tag{5}$$

where $*$ denotes reversing the time component of the quaternion-valued q^μ. The most general form of the non-Abelian component of the electromagnetic field tensor in conformally curved spacetime is [1]

$$F^{\mu\nu} = \frac{1}{8} QR \left(q^\mu q^{\nu *} - q^\nu q^{\mu *} \right) . \tag{6}$$

If we are considering magnetic flux density components of $F^{\mu\nu}$, then Q has the units of weber and R, the scalar curvature, has the units of inverse square meters. In the flat spacetime limit, $R = 0$, so it is clear that the non-Abelian part of the field tensor Eq. (6), vanishes in special relativity. The complete field tensor $F^{\mu\nu}$ vanishes [1] in flat spacetime because the curvature tensor vanishes. These considerations amount to a diametric refutation of Maxwell-Heaviside theory and of the idea that the electromagnetic sector of unified field theory is $U(1)$ in symmetry. It is shown in this paper that the simplest symmetry possible for the electromagnetic sector is $O(3)$. Most generally, the Sachs theory is a closed field theory that unifies in principle all four fields: gravitational, electromagnetic, weak, and strong.

In Sec. 2 it is shown that there exist generally covariant four-valued four-vectors which are components of q^μ, and these are used to construct the basic structure of $O(3)$ electrodynamics in terms of single-valued components of the quaternion-valued metric q^μ. The paper contains a discussion of the fact that the Sachs theory can be reduced to $O(3)$ electrodynamics, which is a Yang-Mills theory [2,3], and summarizes the empirical evidence for both the Sachs and $O(3)$ theories of electrodynamics. In other words, empirical evidence is given of the instances where the Maxwell-Heaviside theory fails and where the Sachs and $O(3)$ electrodynamics succeed in describing empirical data from various sources. The paper provides irrefutable proof that the $\mathbf{B}^{(3)}$ field [4] is a physical field of curved spacetime, a field which vanishes in special relativity. Similarly, the structure of $O(3)$ electrodynamics is one of curved spacetime.

2. DERIVATION OF THE STRUCTURE OF O(3) ELECTRODYNAMICS IN TERMS OF METRICS OF THE SACHS THEORY

In Eq. (5) the product $q^\mu q^{\nu *}$ is quaternion-valued and non-commutative, but not anti-symmetric, in the indices μ and ν. The $\mathbf{B}^{(3)}$ field and structure of $O(3)$ electrodynamics must be found from a special case of Eq. (5), showing that $O(3)$ electrodynamics is a Yang-Mills theory [2,3] and also a theory of general relativity. The important conclusion is reached that Yang-Mills theories can be derived from the Sachs theory, which can therefore describe the weak and strong fields in principle within a closed classical field theory [1] derived from Einstein's general relativity. This result is consistent with the fact that all theories of physics must in principle be theories of general relativity.

From Eq. (1) it is possible to write four-valued generally covariant, components such as

$$q_x = \left(q_x^0, q_x^1, q_x^2, q_x^3\right), \tag{7}$$

which in the limit of special relativity reduces to

$$\sigma_x = (0, \sigma_x, 0, 0). \tag{8}$$

Similarly, one can write

$$q_y = \left(q_y^0, q_y^1, q_y^2, q_y^3\right) \rightarrow (0, 0, \sigma_y, 0) \tag{9}$$

and use the property

$$q_x q_y^* - q_y q_x^* \rightarrow \sigma_x \sigma_y - \sigma_y \sigma_x \tag{10}$$

in the limit of special relativity. The only possibility from Eqs. (7) and (9) is that

$$\begin{gathered} q_x^1 q_y^{2*} - q_y^2 q_x^{1*} = \partial i q_2^3, \\ \downarrow \\ \sigma_x \sigma_y - \sigma_y \sigma_x = \partial i \sigma_2, \end{gathered} \tag{11}$$

where q_x^1 is single-valued. In a 2×2 matrix representation it is:

$$q_x^1 = \begin{bmatrix} 0 & q_x^1 \\ q_x^1 & 0 \end{bmatrix} \rightarrow \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{12}$$

Similarly,

$$q_x^{2*} = \begin{bmatrix} 0\partial & iq_y^2 \\ iq_y & 0 \end{bmatrix} \rightarrow \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{13}$$

$$q = \begin{bmatrix} q & 0 \\ 0 & q_z \end{bmatrix} \rightarrow \sigma_z = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}. \tag{14}$$

Therefore there exist cyclic relations with $O(3)$ symmetry:

$$\begin{aligned} q_x^1 q_y^{2*} - q_y^2 q_x^{1*} &= \partial i q_z^3, \\ q_y^2 q_z^{3*} - q_z^3 q_y^{2*} &= \partial i q_x^1, \\ q_z^3 q_x^{1*} - q_x^1 q_z^{3*} &= \partial i q_y^2, \end{aligned} \tag{15}$$

and the structure of $O(3)$ electrodynamics begins to emerge [2,3]. If the space basis is represented by the complex circular ((1),(2),(3)), then Eqs. (15) become,

$$
\begin{aligned}
q_x^{(1)} q_y^{(2)*} - q_y^{(a)} q_x^{(1)*} &= \partial i q_z^{(3)}, \\
q_y^{(2)} q_z^{(3)*} - q_z^{(3)} q_y^{(2)*} &= \partial i q_x^{(1)}, \\
q_z^{(3)} q_x^{(1)*} - q_x^{(1)} q_z^{(3)*} &= \partial i q_y^{(2)}.
\end{aligned}
\tag{16}
$$

These are cyclic relations between single-valued metric field components in the non-Abelian part [Eq. (6)] of the quaternion-valued $F^{\mu\nu}$. Equation (16) can be put in vector form:

$$
\begin{aligned}
\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} &= i\mathbf{q}^{(3)*}, \\
\mathbf{q}^{(2)} \times \mathbf{q}^{(3)} &= i\mathbf{q}^{(1)*}, \\
\mathbf{q}^{(3)} \times \mathbf{q}^{(1)} &= i\mathbf{q}^{(2)*},
\end{aligned}
\tag{17}
$$

where the asterisk denotes ordinary complex conjugation in Eq. (17) and quaternion conjugation in Eq. (16).

Equation (17) contains vector-valued metric fields in the complex space basis ((1),(2),(3)) [2,3]. Specifically, in $O(3)$ electrodynamics, which is based on the existence of two circularly polarized components of electromagnetic radiation [2]:

$$
\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}}(i\mathbf{i} + \mathbf{j})e^{i\phi}, \tag{18}
$$

$$
\mathbf{q}^{(2)} = \frac{1}{\sqrt{2}}(-i\mathbf{i} + \mathbf{j})e^{-i\phi}, \tag{19}
$$

giving

$$
\mathbf{q}^{(3)*} = \mathbf{k} \tag{20}
$$

and

$$
\mathbf{B}^{(3)} = \frac{1}{8} QR\mathbf{q}^{(3)}. \tag{21}
$$

Therefore the $B^{(3)}$ field [4] is proven from a particular choice of metric using the irreducible representations of the Einstein group. It can be seen from Eq. (21) that the $\mathbf{B}^{(3)}$ field is the vector-valued metric field $q^{(3)}$ within a factor $\frac{1}{8}QR$. This result proves that $\mathbf{B}^{(3)}$ vanishes in flat spacetime, because $R = 0$ in flat spacetime. If we write

$$
B^{(0)} = \frac{1}{8} QR, \tag{22}
$$

then Eq. (17) becomes B cyclic theorem [2,3] of $O(3)$ electrodynamics:

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*}, \quad \text{et cyclicum.} \tag{23}$$

Since $O(3)$ electrodynamics is a Yang-Mills theory [2,3], we can write

$$q = q^{(1)}\mathbf{i} + q^{(2)}\mathbf{j} + q^{(3)}\mathbf{k}, \tag{24}$$

from which it follows that

$$D^{\mu}(D_{\mu}\mathbf{q}) = \mathbf{0}, \quad D_{\mu}\mathbf{q} = \mathbf{0}, \tag{25}$$

i.e., the first and second covariant derivatives of q vanish.

DISCUSSION

In this discussion we give empirical evidence for the ability of both the Sachs and $O(3)$ theories to describe data which cannot be described by the Maxwell-Heaviside theory of flat space-time. The Sachs theory is able to describe parity violating and spin-spin interactions from first principles [5] on a classical level; can explain several problems of neutrino physics; the Pauli exclusion principle can be derived from it. The quaternion form of the theory, which is the basis of this paper, was first developed in 1982 [6] and predicts small but non-zero masses for the neutrino and photon; it describes the Planck spectrum of black body radiation classically; describes the Lamb shifts in H; proposes grounds for charge quantization; predicts the lifetime of the muon state; describes the electron-muon mass splitting; predicts physical longitudinal and scalar photons and fields.

To this list we can add many additional advantages of $O(3)$ electrodynamics over $U(1)$ electrodynamics; the following list is not recognizable as a consequence of the fact that in flat spacetime the electromagnetic field vanishes, causing many self-inconsistencies to emerge, for which the simplest remedy is $O(3)$ electrodynamics:

(1) There is a self-inconsistency in the gauge theory that leads to the Maxwell-Heaviside theory in that the former eliminates a commutator such as $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ by definition. This commutator is, however, an observable of the inverse Faraday effect, and is inputted phenomenologically in the Maxwell-Heaviside theory. In $O(3)$ electrodynamics this commutator is defined self-consistently as part of the definition of the field tensor [2] in conformally curved space-time and the commutator is proportional to the $\mathbf{B}^{(3)}$ field, as can be seen in Eq. (23), and in the third Stokes parameter [2,3].

(2) The Sagnac effect with platform at rest cannot be described by Maxwell-Heaviside theory due to motion reversal symmetry [7].

There is no phase shift, contrary to observation. The $O(3)$ electrodynamics succeeds in explaining the phase shift both with platform at rest and in motion, with great precision [7].

(3) The phase shift of the Michelson interferometer cannot be described by Maxwell-Heaviside theory due to parity inversion symmetry, whereas the $O(3)$ electrodynamics succeeds in describing the effect accurately [8].

(4) Normal reflection is not described by Maxwell-Heaviside theory again because of parity inversion symmetry [3]. The $O(3)$ electrodynamics describes the effect precisely through a non-Abelian Stokes Theorem [4,9]. More generally, physical optics is a theory of general relativity, which is why these inconsistencies emerge in the Maxwell-Heaviside theory.

(5) The $O(3)$ field equations are Yang-Mills equations which can now be recognized as special cases of Sachs theory. These are equations of conformally curved spacetime.

(6) The $O(3)$ field equations are homomorphic with Barrett's $SU(2)$ field equations and have been developed extensively [4,9] in close coordination with empirical data.

(7) The $O(3)$ theory gives the correct topological and dynamical phases in interferometry while Maxwell-Heaviside theory fails to describe interferometry and physical optics. The fundamental reason is that the electromagnetic field vanishes in flat space-time, so a theory of conformally curved space-time, such as $O(3)$, is required.

(8) One consequence of the adaptation of an $O(3)$ sector symmetry for electrodynamics is that the electroweak theory becomes $SU(2) \times SU(2)$, with concomitant prediction of an observed massive boson [10,11].

(9) The Lorentz condition is eliminated in $O(3)$ electrodynamics [9], allowing allowing predictions of energy from the vacuum. Electromagnetic energy from curved spacetime is inherent in the Sachs theory [1] and $O(3)$ theory [12].

(10) $O(3)$ electrodynamics have been applied to quantum electrodynamics showing minute corrections to the Lamb shift and anomalous g factor of the electron [4].

(11) No contradiction with empirical data has been found with $O(3)$ electrodynamics, nor with the Sachs theory. The above lists many contradictions with data of the Maxwell-Heaviside theory. The fundamental reason for these contradictions is that if one examines the irreducible representations of the Einstein group, the electromagnetic field vanishes in flat spacetime.

(12) The $O(3)$ electrodynamical structure is mathematically that of a Yang-Mills theory with a physical internal gauge space based on the existence of circular polarization and labeled ((1),(2),(3)) as in Sec. 2 of this paper.

(13) There is a self-inconsistency in the stress-energy-momentum tensor of the Maxwell Heaviside theory which is removed

by the $O(3)$ theory [13].

(14) The $O(3)$ theory saves the correspondence principle in the Compton effect [9] whereas the $U(1)$ theory fails to do so.

(15) The technique of radiatively induced fermion resonance is predicted in $O(3)$ electrodynamics [3,4] and is corroborated in the paramagnetic inverse Faraday effect [14]. Neither effect exists in Maxwell-Heaviside theory without additional phenomenology.

(16) The $O(3)$ field equations produce observed soliton and instanton solutions [4] which are missing from the Maxwell-Heaviside equations.

(17) The $O(3)$ electrodynamics lead to the Crowell duality principle [4], an example of which is the $SU(2) \times SU(2)$ structure of electroweak theory.

(18) Covariant $O(3)$ derivatives are used in $O(3)$ theory, indicating that it is a theory of conformally curved spacetime.

In summary, by interlocking the Sachs and $O(3)$ theories, it becomes apparent that the advantages of $O(3)$ over $U(1)$ are symptomatic of the fact that the electromagnetic field vanishes in flat space-time (special relativity), if the irreducible representations of the Einstein group are used as described by Sachs [1].

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