CHAPTER THREE : ECE THEORY AND BELTRAMI FIELDS

3.1 INTRODUCTION

Towards the end of the nineteenth century the Italian mathematician Eugenio Beltrami developed a system of equations for the description of hydrodynamic flow in which the curl of a vector is proportional to the vector itself. An example is the use of the velocity vector. For a long time this solution was not used outside the field of hydrodynamics, but in the fifties it started to be used by workers such as Alfven and Chandrasekhar in the area of cosmology, notably whirlpool galaxies. The Beltrami field as it came to be known has been observed in plasma vortices and as argued by Reed {7} is indicative of a type of electrodynamics such as ECE. Therefore this chapter is concerned with the ways in which ECE electrodynamics reduce to Beltrami electrodynamics, and with other applications of the Beltrami electrodynamics such as a new theory of the parton structure of elementary particles. The ECE theory is based on geometry and is ubiquitous throughout nature on all scales, and so is the Beltrami theory, which can be looked upon as a sub theory of ECE theory.

3.2 DERIVATION OF THE BELTRAMI EQUATION

Consider the Cartan identity in vector notation, derived in chapter two:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{v}^b = \underline{v}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_c - \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{v}^b . \quad -(1)$$

In the absence of a magnetic monopole:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{v}^b = 0 \quad -(2)$$

so:

$$\underline{q}^{b} \cdot \underline{\nabla} \times \underline{\omega}^{a}{}_{b} = \underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} \times \underline{q}^{b} \,. \quad -(3)$$

Assume that the spin connection is an axial vector dual in its index space to an antisymmetric tensor:

$$\underline{\omega}^{a}{}_{b} = \epsilon^{a}{}_{bc}\, \underline{\omega}^{c} \quad -(4)$$

where $\epsilon^{a}{}_{bc}$ is the totally antisymmetric unit tensor in three dimensions. Then Eq. (3) reduces to:

$$\underline{q}^{b} \cdot \underline{\nabla} \times \underline{\omega}^{c} = \underline{\omega}^{c} \cdot \underline{\nabla} \times \underline{q}^{b} \,. \quad -(5)$$

An example of this in electromagnetism is:

$$\underline{A}^{(2)} \cdot \underline{\nabla} \times \underline{\omega}^{(1)} = \underline{\omega}^{(1)} \cdot \underline{\nabla} \times \underline{A}^{(2)} \quad -(6)$$

in the complex circular basis ((1), (2), (3)). The vector potential is defined by the ECE hypothesis:

$$\underline{A}^{a} = A^{(0)}\, \underline{q}^{a} \,. \quad -(7)$$

From chapter two, the geometrical condition for the absence of a magnetic monopole is:

$$\underline{\omega}^{a}{}_{b} \cdot \underline{B}^{b} = \underline{A}^{b} \cdot \underline{R}^{a}{}_{b}(spin) \quad -(8)$$

where the spin curvature is defined by:

$$\underline{R}^{a}{}_{b}(spin) = \underline{\nabla} \times \underline{\omega}^{a}{}_{b} - \underline{\omega}^{a}{}_{c} \times \underline{\omega}^{c}{}_{b} \quad -(9)$$

and where $\underline{B}^{a}$ is the magnetic flux density vector. Using Eq. (4):

$$\underline{R}^{c}(\text{spin}) = \underline{\nabla} \times \underline{\omega}^{c} - \underline{\omega}^{b} \times \underline{\omega}^{a}. \quad -(10)$$

In the complex circular basis defined by Eq. (6) the spin curvatures are:

$$\left.\begin{aligned} \underline{R}^{(1)}(\text{spin}) &= \underline{\nabla} \times \underline{\omega}^{(1)} + i\,\underline{\omega}^{(3)} \times \underline{\omega}^{(1)} \\ \underline{R}^{(2)}(\text{spin}) &= \underline{\nabla} \times \underline{\omega}^{(2)} + i\,\underline{\omega}^{(2)} \times \underline{\omega}^{(3)} \\ \underline{R}^{(3)}(\text{spin}) &= \underline{\nabla} \times \underline{\omega}^{(3)} + i\,\underline{\omega}^{(1)} \times \underline{\omega}^{(2)} \end{aligned}\right] \quad -(11)$$

and the magnetic flux density vectors are:

$$\left.\begin{aligned} \underline{B}^{(1)} &= \underline{\nabla} \times \underline{A}^{(1)} + i\,\underline{\omega}^{(3)} \times \underline{A}^{(1)} \\ \underline{B}^{(2)} &= \underline{\nabla} \times \underline{A}^{(2)} + i\,\underline{\omega}^{(2)} \times \underline{A}^{(3)} \\ \underline{B}^{(3)} &= \underline{\nabla} \times \underline{A}^{(3)} + i\,\underline{\omega}^{(1)} \times \underline{A}^{(2)} \end{aligned}\right] \quad -(12)$$

Eq. (8) may be exemplified by:

$$\underline{\omega}^{(1)} \cdot \underline{B}^{(2)} = \underline{A}^{(1)} \cdot \underline{R}^{(2)}(\text{spin}) \quad -(13)$$

which may be developed as:

$$\begin{aligned} &\underline{\omega}^{(1)} \cdot \left(\underline{\nabla} \times \underline{A}^{(2)} + i\,\underline{\omega}^{(2)} \times \underline{A}^{(3)}\right) \\ &= \underline{A}^{(1)} \cdot \left(\underline{\nabla} \times \underline{\omega}^{(2)} + i\,\underline{\omega}^{(2)} \times \underline{\omega}^{(3)}\right). \end{aligned} \quad -(14)$$

Possible solutions are

$$\underline{\omega}^{(i)} = \pm \frac{\kappa}{A^{(0)}} \underline{A}^{(i)}, \quad i = 1, 2, 3 \quad -(15)$$

and in order to be consistent with the original {1 - 10} solution of B(3) the negative sign is developed:

$$\underline{B}^{(3)} = \underline{\nabla} \times \underline{A}^{(3)} - i\frac{\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)}. \quad -(16)$$

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From Eq. (2):

$$\underline{\nabla} \cdot \underline{\omega}^{(3)} \times \underline{A}^{(1)} = 0 \quad -(17)$$

and the following is an identity of vector analysis:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{A}^{(1)} = 0. \quad -(18)$$

A possible solution of Eq. (17) is:

$$\underline{\nabla} \times \underline{A}^{(1)} = i\underline{\omega}^{(3)} \times \underline{A}^{(1)} = -\frac{i\kappa}{A^{(0)}} \underline{A}^{(3)} \times \underline{A}^{(1)}. \quad -(19)$$

Similarly:

$$\underline{\nabla} \times \underline{A}^{(2)} = i\underline{\omega}^{(2)} \times \underline{A}^{(3)} = -\frac{i\kappa}{A^{(0)}} \underline{A}^{(2)} \times \underline{A}^{(3)}. \quad -(20)$$

Now multiply both sides of the basis equations (6) to (8) of chapter two by

$$A^{(0)2} e^{i\phi} e^{-i\phi} \quad -(21)$$

where the electromagnetic phase is:

$$\phi = \omega t - \kappa Z \quad -(22)$$

to find the cyclic equation:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = iA^{(0)} \underline{A}^{(3)*} \quad -(23)$$

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where:

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = A^{(0)} \underline{e}^{(1)} e^{i\phi} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi}, \quad -(24)$$

$$\underline{A}^{(3)} = A^{(0)} \underline{e}^{(3)} = A^{(0)} \underline{k}. \quad -(25)$$

From Eqs. (23), (24) and (25):

$$\underline{\nabla} \times \underline{A}^{(1)} = \kappa \underline{A}^{(1)} \quad -(26)$$

$$\underline{\nabla} \times \underline{A}^{(2)} = \kappa \underline{A}^{(2)} \quad -(27)$$

$$\underline{\nabla} \times \underline{A}^{(3)} = 0 \underline{A}^{(3)} \quad -(28)$$

which are Beltrami equations {7}.

The foregoing analysis may be simplified by considering only one component out of the two conjugate components labelled (1) and (2). This procedure, however, loses information in general. By considering one component, Eq. (1) is simplified to:

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{v} = \underline{v} \cdot \underline{\nabla} \times \underline{\omega} - \underline{\omega} \cdot \underline{\nabla} \times \underline{v} \quad -(29)$$

and the assumption of zero magnetic monopole leads to:

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{v} = 0 \quad -(30)$$

which implies

$$\underline{\omega} \cdot \underline{\nabla} \times \underline{v} = \underline{v} \cdot \underline{\nabla} \times \underline{\omega} \quad -(31)$$

proceeding as in note 257(7) in the UFT section of www.aias.us leads to:

$$\underline{\omega} \cdot \underline{B} = \underline{A} \cdot \underline{\nabla} \times \underline{\omega} \quad -(32)$$

where:

$$\underline{R}(\text{spin}) = \underline{\nabla} \times \underline{\omega} \quad -(33)$$

is the simplified format of the spin curvature. From Eqs. (31) and (32):

$$\underline{\omega} \cdot \underline{B} = \underline{A} \cdot \underline{\nabla} \times \underline{\omega} = \underline{\omega} \cdot \underline{\nabla} \times \underline{A} \quad -(34)$$

so:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(35)$$

However, in ECE theory:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(36)$$

so Eqs. (35) and (36) imply:

$$\underline{\omega} \times \underline{A} = \underline{0}. \quad -(37)$$

Therefore in this simplified model the spin connection vector is parallel to the vector potential. These results are consistent with {1 - 10}:

$$p^\mu = eA^\mu = \hbar \kappa^\mu = \hbar \omega^\mu \quad -(38)$$

from the minimal prescription. So in this simplified model:

$$\omega^\mu = (\omega_0, \underline{\omega}) = \frac{e}{\hbar} A^\mu = \frac{e}{\hbar} (A_0, \underline{A}) \quad -(39)$$

The electric field strength is defined in the simplified model by:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} - c\omega_0 \underline{A} + \phi \underline{\omega} \quad -(40)$$

where the scalar potential is

$$\phi = cA_0. \quad -(41)$$

From Eqs. (39) and (40):

$$\underline{E} = -\underline{\nabla}\phi - \partial \underline{A} / \partial t, \quad -(42)$$

$$\underline{B} = \underline{\nabla} \times \underline{A}, \quad -(43)$$

which is the same as the structure given by Heaviside, but these equations have been derived from general relativity and Cartan geometry, whereas the Heaviside structure is empirical. The equations (29) to (43) are oversimplified however because they are derived by consideration of only one out of two conjugate conjugates (1) and (2). Therefore they are derived using real algebra instead of complex algebra. They lose the B(3) field and also spin connection resonance, developed later in this book.

In the case of field matter interaction the electric field strength $\underline{E}$ is replaced by the electric displacement $\underline{D}$, and the magnetic flux density $\underline{B}$ by the magnetic field strength $\underline{H}$:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P}, \quad -(44)$$
$$\underline{H} = \frac{1}{\mu_0}\left(\underline{B} - \underline{M}\right), \quad -(45)$$

where $\underline{P}$ is the polarization, $\underline{M}$ is the magnetization, ϵ_0 is the vacuum permittivity and μ_0 is the vacuum permeability. The four equations of electrodynamics for each index (1) or (2) are:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(46)$$
$$\underline{\nabla} \times \underline{E} + \partial \underline{B} / \partial t = \underline{0} \quad -(47)$$
$$\underline{\nabla} \cdot \underline{D} = \rho \quad -(48)$$
$$\underline{\nabla} \times \underline{H} = \underline{J} + \partial \underline{D} / \partial t \quad -(49)$$

where ρ is the charge density and $\underline{J}$ is the current density.

The Gauss law of magnetism:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(50)$$

implies the magnetic Beltrami equation {7}:

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B} \quad -(51)$$

because:

$$\frac{1}{\kappa} \underline{\nabla} \cdot \underline{\nabla} \times \underline{B} = 0. \quad -(52)$$

So the magnetic Beltrami equation is a consequence of the absence of a magnetic monopole and the Beltrami solution is always a valid solution. From Eqs. (49) and (51)

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \quad -(53)$$

and for magnetostatics or if the Maxwell displacement current is small:

$$\underline{B} = \frac{\mu_0}{\kappa} \underline{J} . \quad -(54)$$

In this case the magnetic flux density is proportional to the current density. From Eq. (51):

$$\underline{\nabla} \times \underline{B} = \frac{\mu_0}{\kappa} \underline{\nabla} \times \underline{J} = \kappa \underline{B} \quad -(55)$$

so:

$$\underline{B} = \frac{\mu_0}{\kappa^2} \underline{\nabla} \times \underline{J} . \quad -(56)$$

Eqs. (54) and (56) imply that the current density must have the structure:

$$\underline{\nabla} \times \underline{J} = \kappa \underline{J} \quad -(57)$$

in order to produce a Beltrami equation (51) in magnetostatics. Eq. (54) suggests that the jet observed from the plane of a whirlpool galaxy is a longitudinal solution of the Beltrami equation, a J(3) current associated with a B(3) field.

In field matter interaction the electric Beltrami equation:

$$\underline{\nabla} \times \underline{E} = \kappa \underline{E} \quad -(58)$$

is not valid because it is not consistent with the Coulomb law:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 . \quad -(59)$$

From Eqs. (58) and (59):

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{E} = \frac{\rho}{\epsilon_0} \kappa \quad -(60)$$

which violates the vector identity:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{E} = 0 \quad -(61)$$

The electric Beltrami equation:

$$\underline{\nabla} \times \underline{E} = \kappa \underline{E} \quad -(62)$$

is valid for the free electromagnetic field.

Consider the four equations of the free electromagnetic field:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(63)$$
$$\underline{\nabla} \times \underline{E} + \partial \underline{B} / \partial t = \underline{0} \quad -(64)$$
$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(65)$$
$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(66)$$

for each index of the complex circular basis. It follows from Eqs. (64) and (66) that:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{B}) = \frac{1}{c^2}\frac{\partial}{\partial t}\underline{\nabla} \times \underline{E} \quad -(67)$$

and:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{E}) = -\frac{\partial}{\partial t}\underline{\nabla} \times \underline{B} \quad -(68)$$

The transverse plane wave solutions are:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i\phi} \quad -(69)$$

and

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}}(i\underline{i} + \underline{j})e^{i\phi} \quad -(70)$$

where:

$$\phi = \omega t - \kappa Z \quad -(71)$$

and where ω is the angular velocity at instant t and κ is the magnitude of the wave vector at Z.

From vector analysis:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{B}) = \underline{\nabla} (\underline{\nabla} \cdot \underline{B}) - \nabla^2 \underline{B} \quad -(72)$$

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{E}) = \underline{\nabla} (\underline{\nabla} \cdot \underline{E}) - \nabla^2 \underline{E} \quad -(73)$$

and for the free field the divergences vanish, so we obtain the Helmholtz wave equations:

$$(\nabla^2 + \kappa^2)\underline{B} = \underline{0} \quad -(74)$$

and

$$(\nabla^2 + \kappa^2)\underline{E} = \underline{0}. \quad -(75)$$

These are the Trkalian equations:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{B}) = \kappa \underline{\nabla} \times \underline{B} = \kappa^2 \underline{B} \quad -(76)$$

and

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{E}) = \kappa \underline{\nabla} \times \underline{E} = \kappa^2 \underline{E} \quad -(77)$$

So solutions of the Beltrami equations are also solutions of the Helmholtz wave equations.

From Eqs. (64), (67) and (76):

$$-\nabla^2 \underline{B} - \frac{\kappa}{c^2} \frac{\partial \underline{E}}{\partial t} = \left(-\nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \underline{B} = \underline{0} \quad -(78)$$

which is the d'Alembert equation:

$$\square \underline{B} = \underline{0}. \quad -(79)$$

For finite photon mass, implied by the longitudinal solutions of the free electromagnetic field:

$$\hbar^2 \omega^2 = c^2 \hbar^2 \kappa^2 + m_0^2 c^4 \quad -(80)$$

in which case:

$$\left(\Box + \left(\frac{m_0 c}{\hbar} \right)^2 \right) \underline{B} = \underline{0} \quad -(81)$$

which is the Proca equation. This was first derived in ECE theory from the tetrad postulate of Cartan geometry and is discussed later in this book. From Eqs. (67) and (68):

$$\frac{\partial^2}{\partial t^2} \underline{\nabla} \times \underline{B} = -\omega^2 \underline{\nabla} \times \underline{B} \quad -(82)$$

and:

$$\frac{\partial^2}{\partial t^2} \underline{\nabla} \times \underline{E} = -\omega^2 \underline{\nabla} \times \underline{E} . \quad -(83)$$

In general:

$$\frac{\partial^2}{\partial t^2} e^{i\phi} = -\omega^2 e^{i\phi} \quad -(84)$$

and

$$e^{i\phi} = e^{i\omega t} e^{-i\kappa Z} \quad -(85)$$

so the general solution of the Beltrami equation

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B} \quad -(86)$$

will also be a general solution of the equations (63) to (66) multiplied by the phase factor $\exp(i\omega t)$.

ECE theory can be used to show that the magnetic flux density, vector potential and spin connection vector are always Beltrami vectors with intricate structures in general, solutions of the Beltrami equation. The Beltrami structure of the vector potential is proven in ECE physics from the Beltrami structure of the magnetic flux density $\underline{B}$. The space part of the Cartan identity also has a Beltrami structure. If real algebra is use, the Beltrami structure of $\underline{B}$ immediately refutes U(1) gauge invariance because $\underline{B}$ becomes directly proportional to $\underline{A}$. It follows that the photon mass is identically non-zero, however tiny in magnitude. Therefore there is no Higgs boson in nature because the latter is the result of U(1) gauge invariance. The Beltrami structure of $\underline{B}$ is the direct result of the Gauss law of magnetism and the absence of a magnetic monopole. It is difficult to conceive why U(1) gauge invariance should ever have been adopted as a theory, because its refutation is trivial. Once U(1) gauge invariance is discarded a rich panoply of new ideas and results emerge.

The Beltrami equation for magnetic flux density in ECE physics is:

$$\underline{\nabla} \times \underline{B}^a = \kappa \underline{B}^a \quad -(87)$$

In the simplest case κ is a wave-vector but it can become very intricate. Combining Eq. (87) with the Ampere Maxwell law of ECE physics:

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}^a + \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} \quad -(88)$$

the magnetic flux density is given directly by:

$$\underline{B}^a = \frac{1}{\kappa} \left(\frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} + \mu_0 \underline{J}^a \right) \quad -(89)$$

Using the Coulomb law of ECE physics:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(90)$$

it is found that:

$$\underline{\nabla} \cdot \underline{B}^a = \frac{\mu_0}{\kappa} \left(\frac{\partial \rho^a}{\partial t} + \underline{\nabla} \cdot \underline{J}^a \right) = 0 \quad -(91)$$

a result which follows from:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(92)$$

where c is the universal constant known as the vacuum speed of light. The conservation of charge current density in ECE physics is:

$$\frac{\partial \rho^a}{\partial t} + \underline{\nabla} \cdot \underline{J}^a = 0 \quad -(93)$$

so $\underline{B}^a$ is always a Beltrami vector.

In the simplified physics with real algebra:

$$\underline{B} = \underline{\nabla} \times \underline{A}, \quad -(94)$$

$$\underline{\nabla} \times \underline{B} = \kappa \, \underline{\nabla} \times \underline{A} \quad -(95)$$

where $\underline{A}$ is the vector potential. Eqs. (94) and (95) show immediately that in U(1) physics the vector potential also obeys a Beltrami equation:

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A}, \quad -(96)$$

$$\underline{B} = \kappa \underline{A} \quad -(97)$$

so in this simplified theory the magnetic flux density is directly proportional to the vector potential $\underline{A}$. It follows immediately that $\underline{A}$ cannot be U(1) gauge invariant because U(1) gauge invariance means :

$$\underline{A} \rightarrow \underline{A} + \underline{\nabla} \psi \quad -(98)$$

and if $\underline{A}$ is changed, $\underline{B}$ is changed. The obsolete dogma of U(1) physics asserted that Eq. (98)) does not change any physical quantity. This dogma is obviously incorrect because $\underline{B}$ is a physical quantity and Eq. (97) changes it. Therefore there is finite photon mass and no Higgs boson.

Finite photon mass and the Proca equation are developed later in this book, and the theory is summarized here for ease of reference. The Proca equation {1 - 10} can be developed as:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(99)$$

$$\underline{\nabla} \times \underline{E}^a + \partial \underline{B}^a / \partial t = \underline{0} \quad -(100)$$

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(101)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(102)$$

where the four-current density is:

$$J^{a\mu} = (c\rho^a, \underline{J}^a) \quad -(103)$$

and where the four-potential is:

$$A^{a\mu} = \left(\frac{\phi^a}{c}, \underline{A}^a\right). \quad -(104)$$

Proca theory asserts that:

$$J^{a\mu} = -\epsilon_0 \left(\frac{mc}{\hbar}\right)^2 A^{a\mu} \quad -(105)$$

where m is the finite photon mass and $\hbar$ is the reduced Planck constant. Therefore:

$$\rho^a = -\epsilon_0 c^2 (mc/\hbar)^2 \phi^a \quad -(106)$$

$$\underline{J}^a = -\epsilon_0 (mc/\hbar)^2 \underline{A}^a \quad -(107)$$

The Proca equation was inferred in the mid thirties but is almost entirely absent from the textbooks. This is an unfortunate result of incorrect dogma, that the photon mass is zero despite being postulated by Einstein in about 1905 to be a particle of corpuscle, as did Newton before him. The U(1) Proca theory in S. I. Units is:

$$\partial_\mu F^{\mu\nu} = \frac{J^\nu}{\epsilon_0} = -\left(\frac{mc}{\hbar}\right)^2 A^\nu \quad -(108)$$

It follows immediately that:

$$\partial_\nu \partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} \partial_\nu J^\nu = -\left(\frac{mc}{\hbar}\right)^2 \partial_\nu A^\nu = 0 \qquad -(109)$$

and that:

$$\partial_\mu J^\mu = \partial_\mu A^\mu = 0. \qquad -(110)$$

Eq. (110a) is conservation of charge current density and Eq. (110b) is the Lorenz condition. In the Proca equation the Lorenz condition has nothing to do with gauge invariance. The U(1) gauge invariance means that:

$$A^\mu \rightarrow A^\mu + \partial^\mu \chi \qquad -(111)$$

and from Eq. (108) it is trivially apparent that the Proca field and charge current density change under transformation (111), so are not gauge invariant, Q. E. D. The entire edifice of U(1) electrodynamics collapses as soon as photon mass is considered.

In vector notation Eq. (109) is:

$$\frac{1}{c} \frac{\partial}{\partial t} \nabla \cdot \mathbf{E} = \frac{1}{c\epsilon_0} \frac{\partial \rho}{\partial t} = 0 \qquad -(112)$$

and

$$\nabla \cdot \nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \nabla \cdot \mathbf{E} = \mu_0 \nabla \cdot \mathbf{J} = 0 \qquad -(113)$$

Now use:

$$\nabla \cdot \nabla \times \mathbf{B} = 0 \qquad -(114)$$

and the Coulomb law of this simplified theory (without index a):

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \qquad -(115)$$

to find that:

$$-\frac{1}{c^2\epsilon_0}\frac{\partial\rho}{\partial t} = \mu_0 \underline{\nabla}\cdot\underline{J} \quad -(116)$$

which is the equation of charge current conservation:

$$\frac{\partial\rho}{\partial t} + \underline{\nabla}\cdot\underline{J} = 0. \quad -(117)$$

In the Proca theory, Eq. (110) implies the Lorenz gauge as it is known in standard physics:

$$\partial_\mu A^\mu = \frac{1}{c^2}\frac{\partial\phi}{\partial t} + \underline{\nabla}\cdot\underline{A} = 0. \quad -(118)$$

The Proca wave equation in the usual development {13} is obtained from the U(1) definition of the field tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad -(119)$$

so

$$\partial_\mu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) = \Box A^\nu - \partial^\nu\partial_\mu A^\mu = -\left(mc/\hbar\right)^2 A^\nu \quad -(120)$$

in which

$$\partial_\mu A^\mu = 0. \quad -(121)$$

Eq. (121) follows from Eq. (108) in Proca physics, but in standard U(1) physics with identically zero photon mass the Lorenz gauge has to be assumed, and is arbitrary. So the Proca wave equation in the usual development {13} is:

$$\left(\Box + \left(mc/\hbar\right)^2\right)A^\nu = 0. \quad -(122)$$

In ECE physics {1 - 10} Eq. (122) is derived from the tetrad postulate of Cartan geometry and becomes:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) A^a_\mu = 0. \quad -(123)$$

In ECE physics the conservtaion of charge current density is:

$$\partial_\mu J^{a\mu} = 0 \quad -(124)$$

and is consistent with Eqs. (48) and (49).