The parity operator P acts on the fermion spinor as follows:

$$\underline{P}\psi = \begin{bmatrix} \psi_1^L & \psi_2^L \\ \psi_1^R & \psi_2^R \end{bmatrix} \quad -(37)$$

and the anti fermion is obtained straightforwardly from the fermion equation by operating on each term with P as follows:

$$\underline{P}(E) = E, \quad \underline{P}(\underline{p}) = -\underline{p}, \quad \underline{P}\begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} = \begin{bmatrix} \psi_1^L & \psi_2^L \\ \psi_1^R & \psi_2^R \end{bmatrix}. \quad -(38)$$

Note carefully that the eigenstates of energy are always positive, both in the fermion and anti fermion equations. The anti fermion is obtained from the fermion by reversing helicity:

$$\underline{P}(\underline{\sigma}\cdot\underline{p}) = -\underline{\sigma}\cdot\underline{p} \quad -(39)$$

and has opposite parity to the fermion, the same mass as the fermion, and the opposite electric charge. The static fermion is indistinguishable from the static anti fermion {13}. So CPT symmetry is conserved as follows form fermion to anti fermion:

$$C\underline{P}T \rightarrow (-C)\,(-\underline{P}) \quad -(40)$$

where C is the charge conjugation operator and T the motion reversal operator. Note carefully that there is no negative energy anywhere in the analysis.

The pair of simultaneous equations (28) and (29) can be written as:

$$(E - c\underline{\sigma}\cdot\underline{p})(E + c\underline{\sigma}\cdot\underline{p})\phi^L = m^2c^4\phi^L \quad -(41)$$

an equation which can be re arranged as:

$$\left(E^2 - m^2c^4\right)\phi^L = c^2\,\underline{\sigma}\cdot\underline{p}\;\underline{\sigma}\cdot\underline{p}\;\phi^L \quad -\left(\overset{42}{\cancel{35}}\right)$$

and factorized to give:

$$\left(E - mc^2\right)\left(E + mc^2\right)\phi^L = c^2\,\underline{\sigma}\cdot\underline{p}\;\underline{\sigma}\cdot\underline{p}\;\phi^L \quad -\left(\overset{43}{\cancel{36}}\right)$$

If $\underline{p}$ is real valued, Pauli algebra means that:

$$\underline{\sigma}\cdot\underline{p}\;\underline{\sigma}\cdot\underline{p} = p^2 \quad -\left(\overset{44}{\cancel{37}}\right)$$

so if E and $\underline{p}$ are regarded as functions, not operators, Eq. ($\overset{43}{\cancel{38}}$) becomes the Einstein energy equation:

$$E^2 - m^2c^4 = c^2p^2 \quad -\left(\overset{45}{\cancel{38}}\right)$$

multiplied by ϕ^L on both sides. It si well known {1 - 10} that the Einstein energy equation is a way of writing the relativistic energy and momentum:

$$E = \gamma mc^2, \quad -\left(\overset{46}{\cancel{39}}\right)$$

$$\underline{p} = \gamma m\underline{v}. \quad -\left(\overset{47}{\cancel{40}}\right)$$

Realizing this, Eq. ($\overset{43}{\cancel{36}}$) can be linearized as follows. First, express it as:

$$\left(E - mc^2\right)\phi^L = \frac{c^2\,\underline{\sigma}\cdot\underline{p}\;\underline{\sigma}\cdot\underline{p}}{E + mc^2}\,\phi^L \quad -(48)$$

and approximate the total energy:

$$E = \gamma mc^2 \quad -\left(\overset{49}{\cancel{42}}\right)$$

by the rest energy:

$$E \sim mc^2 \quad - (\cancel{43}\ 50)$$

then Eq. (48) becomes:

$$(E - mc^2)\phi^L = \frac{1}{2m}\,\underline{\sigma}\cdot\underline{p}\ \underline{\sigma}\cdot\underline{p}\ \phi^L \quad - (\cancel{43}\ 51)$$

which has the structure of the free particle Schroedinger equation:

$$E_{NR}\,\phi^L = \frac{p^2}{2m}\phi^L \quad - (\cancel{44}\ 52)$$

in which the non relativistic limit of the kinetic energy is defined in the limit v << c by:

$$E_{NR} = E - mc^2 = (\gamma - 1)mc^2 \rightarrow \frac{p^2}{2m}. \quad - (\cancel{45}\ 53)$$

So the fermion equation reduces correctly to the non relativistic Schroedinger equation for the free particle, Q. E. D.

The great importance of the fermion equation to chemical physics emerges from the fact that it can describe the phenomena for which the Dirac equation is justly famous while at the same time eliminating the problem of negative energy as we have just seen. In quantum field theory this leads to a free fermion quantum field theory. This aim is very difficult to achieve {13} in the standard quantum field theory because methods have to be devised to deal with the negative energy. The latter is due simply to Dirac's choice of gamma matrices.

The way in which the fermion equation describes the g factor of the electron, the Landé factor, the Thomas factor and Darwin term is described in the following section.

5.2 INTERACTION OF THE ECE FERMION WITH THE ELECTROMAGNETIC FIELD.

The simplest and most powerful way of describing this interaction for each polarization index a of ECE theory is through the minimal prescription

$$p^\mu \rightarrow p^\mu - eA^\mu \quad -(\overset{54}{\cancel{46}})$$

where a negative sign is used {13} because the charge on the electron is -e. Eq. ($\overset{54}{\cancel{46}}$) can be written as:

$$E \rightarrow E - e\phi \quad -(\overset{55}{47})$$

and:

$$\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A} \quad -(\overset{56}{48})$$

Using Eqs. ($\overset{55}{47}$) and ($\overset{56}{48}$) in the Einstein energy equation ($\overset{45}{\cancel{38}}$) gives:

$$(E - e\phi)^2 = c^2(\mathbf{p} - e\mathbf{A})^2 + m^2c^4 \quad -(\overset{57}{49})$$

which can be factorized as follows:

$$(E - e\phi - mc^2)(E - e\phi + mc^2) = c^2(\mathbf{p} - e\mathbf{A})^2 \quad -(\overset{58}{50})$$

and written as:

$$E = mc^2 + e\phi + c^2(\mathbf{p} - e\mathbf{A})(E - e\phi + mc^2)^{-1}(\mathbf{p} - e\mathbf{A}) \quad -(59)$$

in a form ready for quantization. The latter is carried out with:

$$\mathbf{p} \rightarrow -i\hbar\boldsymbol{\nabla} \quad -(\overset{60}{\cancel{52}})$$

and produces many well known effects and new effects of spin robit coupling described in papers of ECE theory such as UFT 248 ff on www.aias.us.

The most famous result of the Dirac equation, and its improved version, the ECE fermion equation, is electron spin resonance, which depends on the use of the Pauli matrices as is very well known. In this section the various intricacies of this famous derivation are explained systematically. Electron spin resonance occurs in the presence of a static magnetic field, so the scalar potential can be omitted from consideration leaving hamiltonians such as:

$$H_2\psi = \frac{1}{2m}\left(\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\right)\psi \qquad -(\overset{61}{\cancel{53}})$$

Note carefully that the operator $\underline{\nabla}$ acts on the wave function, which is denoted ψ for ease of notation. The following type of Pauli algebra:

$$\underline{\sigma}\cdot\underline{\nabla}\,\underline{\sigma}\cdot\underline{W} = \underline{\nabla}\cdot\underline{W} + i\underline{\sigma}\cdot\underline{\nabla}\times\underline{W} \qquad -(\overset{62}{\cancel{54}})$$

leads to:

$$H_2\psi = \frac{1}{2m}\Big(ie\hbar\left(\underline{\nabla}\cdot\underline{A} + i\underline{\sigma}\cdot\underline{\nabla}\times\underline{A}\right) - \hbar^2\left(\nabla^2 + i\underline{\sigma}\cdot\underline{\nabla}\times\underline{\nabla}\right) + e^2\left(A^2 + i\underline{\sigma}\cdot\underline{A}\times\underline{A}\right) + ie\hbar\left(\underline{A}\cdot\underline{\nabla} + i\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}\right)\right)\Big)\psi \qquad -(\overset{63}{\cancel{55}})$$

Assuming that $\underline{A}$ is real valued, then:

$$\underline{A}\times\underline{A} = \underline{0}. \qquad -(\overset{64}{\cancel{56}})$$

Also:

$$\underline{\nabla}\times\underline{\nabla} = \underline{0} \qquad -(\overset{65}{\cancel{57}})$$

so:

$$H_2\phi = \frac{1}{2m}\Big(-\hbar^2\nabla^2\phi + e^2A^2\phi + ie\hbar\,\underline{\nabla}\cdot(\underline{A}\phi) - e\hbar\,\underline{\sigma}\cdot\underline{\nabla}\times(\underline{A}\phi) + ie\hbar\,\underline{A}\cdot\underline{\nabla}\phi - e\hbar\,\underline{\sigma}\cdot\underline{A}\times\underline{\nabla}\phi\Big) \quad -(66)$$

It can be seen that the fermion equation produces many effects in general, all of which are experimentally observable. So it is a very powerful result of geometry and ECE unified field theory. Gravitational effects can be considered through the appropriate minimal prescription as in papers such as UFT 248 ff. Many of these effects remain to be observed.

Electron spin resonance is given by the term:

$$H_2\phi = -\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\left(\underline{\nabla}\times(\underline{A}\phi) + \underline{A}\times\underline{\nabla}\phi\right) + \ldots$$

$$= -\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B} + \ldots \quad -(67)$$

where the standard relation between B and A has been used to illustrate the argument:

$$\underline{B} = \underline{\nabla}\times\underline{A} \quad -(68)$$

In the rigorous ECE theory the spin connection enters into the analysis. A vast new subject area of chemical physics emerges because electron spin resonance (ESR) and nuclear magnetic resonance (NMR) dominate the subjects of chemical physics and analytical chemistry.

Use of a complex valued potential such as that in an electromagnetic field rather than a static magnetic field produces many more effects through the equation:

$$\left((E-e\phi) + c\underline{\sigma}\cdot(\underline{p}-e\underline{A})\right)\left((E-e\phi) - c\underline{\sigma}\cdot(\underline{p}-e\underline{A}^*)\right)\phi^R = m^2c^4\phi^R, \quad -(69)$$

$$\phi := \phi^R,$$

i.e.

$$\left(E - e\phi - mc^2\right)\left(E - e\phi + mc^2\right)\phi$$
$$= c^2 \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}\right) \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}^*\right)\phi \quad -(69b)$$
$$+ ec\left(E - e\phi\right) \underline{\sigma} \cdot \left(\underline{A}^* - \underline{A}\right)\phi$$

where * denotes "complex conjugate". Eq. (69b) can be linearized as:

$$\left(E - e\phi - mc^2\right)\phi = \left(\frac{c^2 \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}\right)\underline{\sigma} \cdot \left(\underline{p} - e\underline{A}^*\right)}{E - e\phi + mc^2}\right)\phi + \left(\frac{ec\left(E - e\phi\right)}{E - e\phi + mc^2}\right) \underline{\sigma} \cdot \left(\underline{A}^* - \underline{A}\right)\phi \quad - \cancel{(62)}\,(70)$$

and re arranged as follows:

$$E\phi = \left(e\phi + mc^2\right)\phi$$
$$+ \frac{1}{2m} \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}\right)\left(1 - \frac{e\phi}{2mc^2}\right)^{-1} \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}^*\right)\phi \quad \cancel{(63)}$$
$$+ \frac{e}{2mc}\left(mc^2 - e\phi\right)\left(1 - \frac{e\phi}{2mc^2}\right)^{-1} \underline{\sigma} \cdot \left(\underline{A}^* - \underline{A}\right)\phi \quad -(71)$$

In the approximation:

$$e\phi << mc^2. \quad -(\overset{72}{\cancel{64}})$$

Eq. ($\overset{71}{\cancel{63}}$) gives:

$$E\phi = \left(H_1 + H_2 + H_3\right)\phi \quad -(\overset{73}{\cancel{65}})$$

where the three hamiltonians are defined as follows:

$$H_1 = e\phi + mc^2, \quad -(\overset{74}{\cancel{66}})$$

$$H_2 = \frac{1}{2m} \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}\right)\left(1 + \frac{e\phi}{2mc^2}\right) \underline{\sigma} \cdot \left(\underline{p} - e\underline{A}^*\right), \quad -(\overset{75}{\cancel{67}})$$

$$H_3 = \frac{1}{2} ec\left(1 + \frac{e\phi}{2mc^2}\right) \underline{\sigma} \cdot \left(\underline{A}^* - \underline{A}\right), \quad -(\underset{76}{\cancel{68}})$$

leading to many new fermion resonance effects using the electromagnetic field rather than the static magnetic field.

For example the H_2 hamiltonian can be developed as:

$$H_{21}\phi = \frac{1}{2m}\Big(ie\hbar\left(\nabla\cdot\mathbf{A}^* + i\boldsymbol{\sigma}\cdot\nabla\times\mathbf{A}^*\right) - \hbar^2\left(\nabla^2 + i\boldsymbol{\sigma}\cdot\nabla\times\nabla\right) + e^2\left(\mathbf{A}\cdot\mathbf{A}^* + i\boldsymbol{\sigma}\cdot\mathbf{A}\times\mathbf{A}^*\right) + ie\hbar\left(\mathbf{A}\cdot\nabla + i\boldsymbol{\sigma}\cdot(\mathbf{A}\times\nabla)\right)\Big)\phi \quad -(\cancel{69}\ 77)$$

an equation that can be written as:

$$H_{21}\phi = \frac{1}{2m}\Big(ie^2\boldsymbol{\sigma}\cdot\mathbf{A}\times\mathbf{A}^*\phi - e\hbar\boldsymbol{\sigma}\cdot\mathbf{A}\times\nabla\phi - e\hbar\boldsymbol{\sigma}\cdot\nabla\phi\times\mathbf{A}^* - e\hbar\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{A}^*)\phi + \ldots\Big) \quad -(78)$$

giving four out of many terms that can give novel fermion resonance effects. Using for the sake of argument:

$$\mathbf{B}^* = \nabla\times\mathbf{A}^* \quad -(79)$$

then the hamiltonian reduces to:

$$H_{211} = -\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B}^{*} \qquad -\left(\cancel{72}\ 80\right)$$

and a term due to the conjugate product of the electromagnetic field:

$$H_{212} = i\,\frac{e^{2}}{2m}\,\underline{\sigma}\cdot\underline{A}\times\underline{A}^{*} \qquad -\left(81\ \cancel{73}\right)$$

which defines the B(3) field introduced in previous chapters:

$$\underline{B}^{(3)*} = -ig\,\underline{A}\times\underline{A}^{*} = -ig\,\underline{A}^{(1)}\times\underline{A}^{(2)} \qquad -\left(82\ \cancel{74}\right)$$

Eq. (~~73~~ 81) is the hamiltonian that defines radiatively induced fermion resonance (RFR), extensively discussed elsewhere {1 - 10} but derived here in a rigorous way from the fermion equation or chiral representation of the Dirac equation.