and not by discarding the spin connection. So the MH format achieved in this way is still a theory of general relativity, making unification with gravitation possible.

6. 3 DERIVATION OF THE EQUIVALENCE PRINCIPLE FROM ANTI SYMMETRY AND OTHER APPLICATIONS.

The equivalence of inertial and gravitational mass is known as the weak equivalence principle and has been tested experimentally with great precision. In this section the equivalence principle is derived from anti symmetry. It has been shown independently [1 - 10] by Moses , Reed and Evans that any vector field in three dimensions may be expressed as the sum of three vectors:

$$\underline{V} = \underline{V}^{(1)} + \underline{V}^{(2)} + \underline{V}^{(3)} \quad - (110).$$

in the complex circular basis defined earlier in this book. Helmholtz showed in the nineteenth century that any vector field can be written as the sum of two vectors:

$$\underline{V} = \underline{V}_s + \underline{V}_\ell \quad - (111)$$

where:

$$\underline{\nabla} \cdot \underline{V}_s = 0, \quad - (112)$$

$$\underline{\nabla} \times \underline{V}_\ell = \underline{0}. \quad - (113)$$

The use of the complex circular basis extends the Helmholtz equation as follows:

$$\underline{V}_s = \underline{V}^{(1)} + \underline{V}^{(2)} \quad - (114)$$

$$\underline{V}_\ell = \underline{V}^{(3)}. \quad - (115)$$

The most fundamental components are therefore components of $\underline{V}^{(1)}, \underline{V}^{(2)}, \underline{V}^{(3)}$.

Examples of these fundamental components are:

and so on. In the first papers on ECE theory these components were identified as the objects known as tetrads in Cartan geometry. Such an identification had also been made indirectly by Reed. In Cartan's original definition of the tetrad the a index is the upper index of a four dimensional Minkowski spacetime at point P to a four dimensional manifold indexed . Each of the three dimensional vectors defined in Eq. (110) is the space like component of the following four dimensional vectors:

$$V_\mu^{(i)} = \left(V_0^{(i)}, -\mathbf{V}^{(i)} \right) \quad -(116)$$

$$i = 1, 2, 3 .$$

The complete four dimensional vector is the sum of these three vectors:

$$V_\mu = V_\mu^{(1)} + V_\mu^{(2)} + V_\mu^{(3)} . \quad -(117)$$

So there exist three time like components and the complete time like component is their sum:

$$V_0 = V_0^{(1)} + V_0^{(2)} + V_0^{(3)} . \quad -(118)$$

In four dimensions the a index is:

$$a = (0), (1), (2), (3) \quad -(119)$$

so in general there also exists the component $V_0^{(0)}$. These fundamental elements may always be expressed as tetrad elements and defined as a 4 x 4 matrix as follows:

$$X^a = V_\mu^a X^\mu . \quad -(120)$$

It follows that any four dimensional vector can be defined as a scalar valued quantity multiplied by a Cartan tetrad:

$$V_\mu^a = V q_\mu^a . \quad -(121)$$

Therefore Cartan's differential geometry may be applied to any four dimensional vector. Normally it is applied to the tetrad and the first Cartan structure equation defines the Crtan torsion from the tetrad. The latter is the fundamental building block because it consists of fundamental components of the complete vector field. The Heaviside Gibbs vector analysis restricts consideration to $\underline{V}$ only, but the tetrad analysis realizes that $\underline{V}$ has an internal structure.

In four dimensions therefore define the fundamental vectors:

$$V^{(0)}_{\mu} = \left(V^{(0)}_{0}, \underline{0} \right) \quad -(122)$$

$$V^{(i)}_{\mu} = \left(V^{(i)}_{0}, -\underline{V}^{(i)} \right), i = 1, 2, 3 \quad -(123)$$

Eq. (122) means that the space like components of $V^{(0)}_{\mu}$ are zero by definition because the superscript (0) is time like by definition. There are no space like components of a time like property. On the other hand a vector such as $V^{(1)}_{\mu}$ is a four vector, so $V^{(0)}_{0}$ in general is its non-zero time like component. In general the Cartan tetrad is defined by:

$$X^{a} = q^{a}_{\mu} X^{\mu} \quad -(124)$$

where X denotes any vector field. Therefore Cartan geometry extends the Heaviside Gibbs analysis and this finding can be applied systematically to physics, notably dynamics. The Heaviside Gibbs analysis was restricted to three dimensional space with no connection, i.e. a Euclidean space. Using Cartan's differential geometry the analysis can be extended to any space of any dimension by use of the Cartan spin connection. Using this procedure all the equations of physics can be derived automatically within a unified framework, thus producing the first successful unified field theory.

Now apply this method to the concept of velocity in dynamics. The velocity tetrad is:

$$v^a_\mu = v\, q^a_\mu \qquad -(125)$$

where v is the scalar magnitude of velocity, i.e. the speed. The gravitational potential is defined as:

$$\Phi^a_\mu = c\, v^a_\mu = \Phi\, q^a_\mu . \qquad -(126)$$

In analogy the electromagnetic potential is also defined in terms of the tetrad in ECE theory:

$$A^a_\mu = A^{(0)} q^a_\mu \qquad -(127)$$

The electromagnetic field is defined in terms of the Cartan torsion:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \qquad -(128)$$

and also the gravitational field:

$$g^a_{\mu\nu} = \Phi\, T^a_{\mu\nu} . \qquad -(129)$$

The acceleration due to gravity in ECE theory is therefore part of the torsion, so in general the acceleration in electrodynamics is also part of the torsion, defined conveniently as:

$$a^a_{\mu\nu} = c\, v\, T^a_{\mu\nu} . \qquad -(130)$$

In vector notation Eq. (129) splits in to two equations:

$$\underline{a}^a = -\frac{\partial \underline{v}^a}{\partial t} - c\,\underline{\nabla} v^a_0 - c\,\omega^a_{0b}\,\underline{v}^b + c\, v^b_0\, \underline{\omega}^a_b \qquad -(131)$$

and

$$\underline{\Omega}^a = \underline{\nabla} \times \underline{v}^a - \underline{\omega}^a{}_b \times \underline{v}^b . \quad -(132)$$

The spin connection is defined as:

$$\omega^a_{\mu b} = \left(\omega^a{}_{0b} , - \underline{\omega}^a{}_b \right) . \quad -(133)$$

In tensor notation the relation between acceleration and velocity in generally covariant

dynamics is:

$$a^a_{\mu\nu} = c \left(\partial_\mu v^a_\nu - \partial_\nu v^a_\mu + v \left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu} \right) \right) . \quad -(134)$$

Sp Eqs. (131) and (132) may be simplified to:

$$\underline{a}^a = - \frac{\partial \underline{v}^a}{\partial t} + \underline{\nabla} \Phi^a + c v \, \underline{\omega}^a{}_{orb} \quad -(135)$$

and:

$$\underline{\Omega}^a = \underline{\nabla} \times \underline{v}^a + v \, \underline{\omega}^a{}_{spin} \quad -(136)$$

where:

$$\underline{\omega}_{orb} = \left(\omega^a_{01} - \omega^a_{10} \right) \underline{i} + \left(\omega^a_{02} - \omega^a_{20} \right) \underline{j} + \left(\omega^a_{03} - \omega^a_{30} \right) \underline{k} \quad -(137)$$

and

$$\underline{\omega}_{spin} = \left(\omega^a_{32} - \omega^a_{23} \right) \underline{i} + \left(\omega^a_{13} - \omega^a_{31} \right) \underline{j} + \left(\omega^a_{21} - \omega^a_{12} \right) \underline{k} \quad -(138)$$

and where:

$$v \, \underline{\omega}^a{}_{orb} = - \omega^a{}_{0b} \underline{v}^b + v^b_0 \, \underline{\omega}^a{}_b \quad -(139)$$

and

$$v \, \underline{\omega}^a{}_{spin} = - \underline{\omega}^a{}_b \times \underline{v}^b \quad -(140)$$

Equations (139) and (140) are Coriolis type accelerations due to orbital and spin torsion. Eq. (135) shows that acceleration is due to the rate of change of velocity and also the gradient of the potential. If the inertial frame of Newtonian dynamics is defined as flat space time then in the inertial frame:

$$\underline{a}^a = -\frac{\partial \underline{v}^a}{\partial t} - \underline{\nabla}\,\Phi^a \quad -(141)$$

The equivalence principle assumes that:

$$-\frac{\partial \underline{v}^a}{\partial t} = -\underline{\nabla}\,\Phi^a \quad -(142)$$

which is the direct result of the ECE anti symmetry law:

$$\partial_\mu v^a_\nu = -\partial_\nu v^a_\mu \quad -(143)$$

when

$$\mu = 0, \; \nu = 1 \quad -(144)$$

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Force is defined by mass multiplied by acceleration, so

$$\underline{F}^a = -m\frac{\partial \underline{v}^a}{\partial t} = -m\underline{\nabla}\,\Phi^a \quad -(145)$$

which is a generalization of the weak equivalence principle assumed by Newton but not proven by him. ECE theory shows that the equivalence principle has a geometrical origin.