Unfinished notes, MWE, 1st March, 1993

SOME REMARKS ON THE PROCA FIELD EQUATION.

The Proca equation can be written as

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu} = -\xi^2 A_\nu \; ; \; \xi = \frac{m_0 c}{\hbar} \quad -(1)$$

where $F_{\mu\nu}$ is the electromagnetic four-tensor, the four-curl of A_ν :

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} \quad -(2)$$

and m_0 is the photon rest mass. Multiplying both sides of eqn. (1) by A_ν :

$$A_\nu \frac{\partial F_{\mu\nu}}{\partial x_\mu} = -\xi^2 A_\nu A_\nu \quad -(3)$$

It is clear that if $A_\nu A_\nu \rightarrow 0$, then the Proca equation becomes identical with the free space Maxwell equation:

$$A_\nu A_\nu \rightarrow 0 \; ; \; \frac{\partial F_{\mu\nu}}{\partial x_\mu} \rightarrow 0. \quad -(4)$$

This occurs independently of the photon mass m_0. The condition (4) can be obtained from the Dirac gauge as the photon radius becomes infinitesimally small. Under this condition freedom of gauge choice is maintained, since in this limit, Maxwell's equations are recovered.

Therefore, problems encountered in the quantization of the Maxwell equations disappear if we use the Dirac gauge, which in its limiting form becomes:

$$A_\nu A_\nu^* \rightarrow 0 \; ; \; A_\nu \text{ complex} \quad -(5)$$

i.e. $A_\nu A_\nu^*$ is finite but infinitesimally different from zero. In this light the Maxwell equation (4) corresponds in its quantized form to a photon with infinitesimally small radius. This is also the meaning of the gauge condition $A_\mu A_\mu \rightarrow 0$, which is consistent for all practical purposes with conservation of charge and current and with the fundamental requirement of gauge invariance of the second kind, but which is inconsistent with the Coulomb, or transverse, gauge. This inconsistency is due to the fact that from eqn. (4):

$$|\underset{\sim}{A}| \rightarrow \phi / c \; ; \; A_\mu \equiv \left(\underset{\sim}{A}, \frac{i\phi}{c}\right) \quad -(6)$$

i.e. ϕ/c becomes infinitesimally close to $|\underset{\sim}{A}|$ under the limiting condition (4). In the Coulomb gauge, however,

$$\phi = 0 \; ; \; \underset{\sim}{A} \neq \underset{\sim}{0} \quad -(7)$$

and this cannot be consistent with eqn. (6), nor can it be consistent with the Proca equation. By using the gauge condition (5) the Proca equation is seen to be consistent with gauge invariance of the second kind, and to reduce to the Maxwell equation while retaining a finite photon mass. The Maxwell equation under this gauge condition does not necessarily mean zero photon mass.

Longitudinal Fields.

Under the gauge condition (5) longitudinal electric and magnetic field solutions to the Proca equation become infinitesimally different from those of the Maxwell equation. For all practical purposes the solutions from both types of equation are identical. The electromagnetic energy density is described in terms of the equations:

$$E_\mu E_\mu = 0; \quad B_\mu B_\mu = 0 \qquad — (8)$$

where E_μ and B_μ are electric and magnetic field four vectors. There is no contribution to the electromagnetic energy density from the longitudinal components of E_μ and B_μ because these take the Maxwellian limiting form

$$\underline{B}^{(3)} = B^{(0)}\underline{k}; \quad \underline{E}^{(3)} = E^{(0)}\underline{k} \qquad — (9).$$

i.e. are phase independent, solenoidal fields. This is consistent with the conclusions of Bass and Schrödinger, who explained why the longitudinal solutions of the Proca equations do not contribute to the Planck radiation law. It is clear that if energy is proportional to frequency, as in the Planck law, then the longitudinal solutions of the Proca and Maxwell equations in free space can have no Planck energy, because they are zero frequency solutions. This is expressed classically by equations (8).

Physical Meaning of the Four Potential.

The Bohm Aharonov effect shows that A_μ is physically meaningful in quantum theory, and the Proca equation is an equation of the quantized field which produces a particle, the photon, with three spacelike polarizations. This implies that there are three physically meaningful field polarizations. The fact that A_μ is physically meaningful, as shown experimentally, means that all four components of E_μ and B_μ are physically meaningful, and there exist longitudinal fields whose effect should be detectible experimentally. That these fields are frequency independent can be shown from Lie algebras such as

$$\begin{aligned}
\underline{B}^{(1)} \times \underline{B}^{(2)} &= iB^{(0)}\underline{B}^{(3)*} = iB^{(0)}\underline{B}^{(3)} \\
\underline{B}^{(2)} \times \underline{B}^{(3)} &= iB^{(0)}\underline{B}^{(1)*} = iB^{(0)}\underline{B}^{(2)} \\
\underline{B}^{(3)} \times \underline{B}^{(1)} &= iB^{(0)}\underline{B}^{(2)*} = iB^{(0)}\underline{B}^{(1)}.
\end{aligned} \qquad — (10)$$

linking the longitudinal (3) component to the transverse (1) and (2) components in the circular basis. The frequency independence of the (3) fields is consistent, as we have seen, with the fact that they do not contribute to the Planck radiation law, as discussed by Bass and Schrödinger.

Quantization.

It is well known that quantization of the electromagnetic field, regarded as a massless gauge field, is beset with considerable difficulties. If the limiting form (5) of the Dirac gauge is accepted, it becomes logically impossible to accept quantization in the Coulomb gauge, the traditional method, which leads to two, physically meaningful, transverse polarizations, but which loses manifest covariance. We therefore abandon the Coulomb gauge. Quantization of the massless electromagnetic field in the Lorentz gauge is also beset with difficulties which can be traced to the fact that the mass term is zero in the fundamental Lagrangian. Using the condition (4), and accepting that $A_\mu A_\mu$ is very small in magnitude but not identically zero, the Lagrangian becomes:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_0 A_\mu A_\mu \quad — (11)$$

which is that associated with the Proca equation. Standard quantum field theory then shows that quantization based on the lagrangian (11) is straightforward, and leads to a particle, the photon, with three physically meaningful polarizations. This is consistent for all practical purposes with gauge invariance of the second kind provided we accept the limiting form (4) of the Dirac gauge. This form also allows conservation of charge and current for finite photon mass. If $A_\mu A_\mu$ is identically zero, gauge invariance of the second kind is satisfied for finite photon mass, but the Proca equation becomes identical with the Maxwell equation, and the problems of quantizing the latter equation in the Lorentz gauge reappear.

The limiting form (4) of the Dirac gauge is therefore needed for satisfactory quantization of the electromagnetic field, retaining finite photon mass.

In the classical field, however, the Maxwell equations give longitudinal, physically meaningful, fields which are independent of frequency and therefore have no Planck energy. The Proca equation gives an exponentially decaying form for, for example, the longitudinal magnetic flux density:

$$\underline{B}^{(3)} = B^{(0)} \exp(-\xi Z)\, \underline{k} \quad — (12)$$

a form which depends on the photon mass in its rest frame, and from de Broglie's Guiding Theorem, the mass is associated with a frequency, i.e. has a wave nature:

$$\hbar \omega_0 = m_0 c^2 \quad — (13)$$

$$\Rightarrow \underline{B}^{(3)} = B^{(0)} \exp\left(-\frac{2\pi\omega_0}{c} Z\right) \underline{k} \; ; \; \kappa_0 \equiv \frac{2\pi\omega_0}{c}$$

This feature does not appear in the equivalent Maxwellian longitudinal solutions, which, for the magnetic field, takes the form:

$$\underline{B}^{(3)} = B^{(0)}\underline{k}. \qquad — (14)$$

As we have seen, the gauge condition:

$$A_\mu A_\mu \rightarrow 0 \qquad — (15)$$

does not necessarily mean that the photon mass becomes zero, it means that the photon radius becomes infinitesimally small, but not identically zero. We see therefore that the photon mass, although it may be different from zero, does not appear in the Maxwell equations, and solutions thereof. Quantization of the Maxwell equations with $A_\mu A_\mu$ identically equal to zero must fail, therefore, because the photon radius has become identically zero, and therefore there is no photon. This then implies that there can be no photon mass in the Maxwell equation.

The Photon.

The photon must therefore be interpreted as a particle with three physically meaningful polarizations in space, with finite radius, and finite mass. This particle co-exists with a concommitant electromagnetic field with physically meaningful transverse and longitudinal electric and magnetic field components in vacuo. The longitudinal components correspond with zero frequency and zero Planck energy for all practical purposes, although in the rigorous, exponentially decaying, longitudinal solutions of the Proca equation in vacuo the finite photon rest mass leads to a concommitant non-zero frequency through the Guiding Theorem. These longitudinal solutions, are then, associated with a very low frequency:

$$\nu_0 = m_0 c^2 / \hbar \qquad — (16)$$

of about 10^{-23} hertz if m_0 is about 10^{-68} kgm.

Consequences in the Laboratory

There are available many interesting reviews that set precise experimental limits on photon mass, which is no longer set to identically zero in the standard tables. The major change in experimental perception of the photon due to the existence of longitudinal solutions to Maxwell's equations manifests itself in a series of expected effects due to the frequency independent magnetic and electric fields. These are described in the volume "The Photon's Magnetic Field" by M. W. Evans, (World Scientific, 1993). Especially interesting is the expectation of an optical Bohm Aharonov effect due to $\underline{B}^{(3)}$. On the grounds of the above arguments, these effects can be intepreted as proof for finite photon mass.

Note on Barron's Paradox.

Barron has recently suggested that longitudinal solutions of the Proca equation, which have been known for over fifty years, violate C symmetry, essentially because a field such as $\underline{B}^{(3)}$ changes sign with C while the beam helicity does not. This point of view is however, contradicted by the defining Lie algebra, e.g. eqns. (10). The fact that $\underline{B}^{(3)}$ changes sign under C is a simple consequence of the well known negative charge parity of the photon:

$$\hat{C}(\gamma) = -\gamma \; ; \text{ or } \; \hat{C}(A_\mu) = -A_\mu . \quad -(17)$$

This negative charge parity means that $B^{(0)}$, the scalar magnetic amplitude of the electromagnetic field in vacuo, changes sign under C, while helicity by definition does not. In the Einstein / de Broglie theory, therefore, C must change the sign of the concommitant electromagnetic field of the photon, but must by definition have no effect on the photon as particle, described by its helicity and mass.

Barron's paradox therefore reduces to a trivial demonstration that the concommitant field of the photon has negative charge parity, while the photon itself* has positive charge parity. The field (wave) and particle continue to co-exist after application of C, but interaction must now take place with anti-matter. Since the earth and solar system have evolved in a state of matter, not anti-matter, it is assumed that matter interacts with photons, not anti-photons. The anti-photon is defined as having the same mass and helicity as the photon, but opposite A_μ.

All good wishes,
Sincerely Yours,

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* i.e. as particle, defined by helicity & mass. One cannot have this particle without its field, by de Broglie's Guiding Theorem.