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OPTICS AND INTERFEROMETRY IN O(3) ELECTRODYNAMICS

by

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ABSTRACT

Optics and interferometry are described by an O(3) invariant electrodynamics, and several advantages shown over the received description based on a U(1) invariant electrodynamics. Therefore electrodynamics under all conditions is in general a non-Abelian gauge field theory as indicated by the irreducible representations of the Einstein group in general relativity.

KEYWORDS: O(3) invariant electrodynamics; optics and interferometry.

1. INTRODUCTION

It has been known since 1982 { 1 } that the irreducible representations of the Einstein group indicate that the field tensor of electrodynamics in curved space-time has a non-Abelian structure under all conditions in general relativity. If there is no space-time curvature there is no electromagnetic field present. The vacuum in this theory { 1} is described as flat space-time in which there is no charge or mass, and therefore there is no electromagnetic field present and so cannot propagate through flat space-time, in which there is no source present. This conclusion refutes the Maxwell Heaviside theory, in which an electromagnetic field is able to propagate through the vacuum in a source free region {2, 3}. The electromagnetic sector therefore cannot be described in gauge theory { 4 } by an Abelian U(1) sector symmetry. In this paper a non-Abelian, O(3) invariant, electrodynamics is shown to describe optical and interferometric effects in general, and it is shown that an Abelian, U(1) invariant, electrodynamics fails to describe optical and interferometric effects in several ways, thus reinforcing the expectations of general relativity described already. These are novel and significant results because the Maxwell Heaviside theory almost universally accepted in the received view {2, 3} is deeply flawed, and the use of a U(1) sector symmetry for electrodynamics is invalidated by general relativity. An O(3) sector symmetry for electrodynamics is correctly non-Abelian, and develops electrodynamics in conformally curved space-time as required by general relativity.

In Section 2 the O(3) invariant phase factor is developed from a non-Abelian Stokes Theorem and shown to describe physical optical effects such as normal reflection, and interferometric effects such as those of Sagnac, Michelson and Young. It is shown that a U(1) invariant electrodynamics fails to describe the Sagnac effect and Michelson interferometry, and that a U(1) invariant electrodynamics fails to describe ordinary normal reflection in physical optics. These are novel and significant indications that electrodynamics is a theory of curved space-time, as indicated by a careful examination { 1 } of the irreducible representations of the Einstein group.

In Section 3 it is shown that a vacuum four-current is always present in an O(3) invariant electrodynamics, as also indicated by general relativity { 1 } . The vacuum four-current acts as a source for the electromagnetic field propagating through the vacuum and also acts as a source therefore of electromagnetic energy from curved space-time, another novel and potentially very useful conclusion. This is discussed further in terms of the general theory of electrodynamics obtained from irreducible representations of the Einstein group.

2. THE PHASE FACTOR IN AN O(3) INVARIANT ELECTRODYNAMICS

The phase factor in an O(3) invariant electrodynamics can be obtained from the non-Abelian Stokes Theorem:

$$\oint D_\mu dx^\mu = -\frac{1}{2}\int [D_\mu, D_\nu] d\sigma^{\mu\nu} \quad -(1)$$

where D_μ is a covariant derivative for any gauge group symmetry:

$$D_\mu := \partial_\mu - igA_\mu \quad -(2)$$

From eqn. (1) we derive the expression:

$$\oint A_\mu dx^\mu = -\frac{1}{2}\int G_{\mu\nu} d\sigma^{\mu\nu} \quad -(3)$$

where A_μ is a vector potential for any gauge group and where $G_{\mu\nu}$ is a field tensor for any gauge group:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \quad -(4)$$

It is readily checked that eqn. (3) reduces to the ordinary Stokes Theorem for a U(1) invariant electrodynamics.

In an O(3) invariant electrodynamics there is an internal gauge space of O(3) symmetry characterised by the complex basis ((I), (2), (3)) { 5 - 8 } and an additional relation:

$$\oint A_Z^{(3)} dZ = -ig \int [A_X^{(1)}, A_Y^{(2)}] \, dAr$$

$$= \int B_Z^{(3)} \, dAr \qquad - (5)$$

where both $\underline{A}^{(3)}$ and $\underline{B}^{(3)}$ are longitudinally directed and are non-zero in the vacuum. Both $\underline{A}^{(3)}$ and $\underline{B}^{(3)}$ are phaseless but propagate with the (1) and (2) components of the field.

If we now define:

$$A^{(0)} = |A_Z^{(3)}| \; ; \; g = \frac{\kappa}{A^{(0)}} \qquad - (6)$$

then an equation is obtained for optics and interferometry:

$$\oint dZ = \kappa \int dAr \qquad - (7)$$

which relates the path integral on the left hand side to the area integral on the right. Multiplying both sides of eqn. (7) by κ gives a relation between the dynamical phase on the left hand side and topological phase on the right hand side:

$$\kappa \oint dZ = \kappa^2 \int dAr \qquad - (8)$$

It can then be shown {9, 10} that the Sagnac effect is produced precisely from a gauge transformation:

$$A_Z^{(3)} \rightarrow A_Z^{(3)} + \frac{1}{g} \partial_Z \Lambda^{(3)} \quad -(9)$$

whereas a U(1) invariant electrodynamics fails to give the Sagnac effect { 9, 10 } essentially because the U(1) phase is T and P invariant.

Optics and interferometry in general are given by the same relation (5) as described elsewhere {9, 10} and again on the U(1) level the phase does not correctly describe optical and interferometric effects again because it is invariant under T and P. Therefore we reach the important conclusion that all optics and interferometry are non-Abelian in nature. This conclusion is reinforced in the next section by considering the O(3) field equations.

3. FIELD EQUATIONS IN O(3) INVARIANT ELECTRODYNAMICS AND IN GENERAL RELATIVITY.

The homogeneous field equation for all gauge group symmetries is a Jacobi identity {11}:

$$\sum_{\sigma,\mu,\nu} [D_\sigma, [D_\mu, D_\nu]] := 0 \quad -(10)$$

whose integral form is:

$$\oint D_\mu dx^\mu + \frac{1}{a_I} [D_\mu, D_\nu] d\sigma^{\mu\nu} = 0. \quad -(11)$$

In O(3) electrodynamics the homogeneous field equation can be written in terms of the dual $G_{\mu\nu}$ of the O(3) field tensor:

$$D_\mu \underline{\tilde{G}}^{\mu\nu} = 0 \quad -(12)$$

i.e. as:

$$\partial_\mu \underline{\tilde{G}}^{\mu\nu} = - \underline{A}_\mu \times \underline{\tilde{G}}^{\mu\nu} \quad -(13)$$

so always contains a vacuum four-current $\left(\underline{J}^\nu := - \underline{A}_\mu \times \underline{\tilde{G}}^{\mu\nu} \right)$. The energy given by this vacuum four-current is then:

$$En = \int \underline{J}^\nu \cdot \underline{A}_\nu \, dV \quad -(14)$$

In the Maxwell Heaviside view on the other hand there is no vacuum four-current, but in general relativity such a current exists { 1 } and has also been introduced empirically by Lehnert { 12 } . The introduction of such a current produces longitudinal modes in vacuo and among these is the $\underset{\sim}{B}^{(3)}$ field { 12 }. The latter cannot therefore be described by a Maxwell Heaviside theory in the vacuum. The development of O(3) electrodynamics is the first stage towards a non-Abelian theory of electrodynamics in curved space-time. Such a theory has been developed by Sachs { 1 } from the irreducible representations of the Einstein group. In curved space-time the electromagnetic field tensor contains longitudinal and transverse components and the metric q^{μ}_{ν} is quaternion valued with sixteen components. This means that the electromagnetic field tensor can be described in general by:

$$F_{\mu\nu} = \partial_\mu A_\nu^* - \partial_\nu A_\mu^* + \frac{1}{8} \Phi^{(0)} R \left(q_\mu q_\nu^* - q_\nu q_\mu^* \right) \quad -(15)$$

where A_μ^* are quaternion valued potentials, $\Phi^{(0)}$ has the units of magnetic flux, and where R is the scalar curvature { 1} . The structure represented by eqn. (15) is non-Abelian, implying that the sector symmetry of electromagnetism in unified field theory cannot be U(1). It must have a non-Abelian symmetry such as O(3) and must be developed in curved space-time. In general all theories of physics must be developed as theories of general relativity. If this is done for electrodynamics { 1 }, the electromagnetic field tensor vanishes if the curvature vanishes, implying that the electromagnetic field cannot propagate through the flat space-time vacuum in which there is no mass or charge present { 1 } . The Maxwell Heaviside theory is therefore an incorrect description of electrodynamics because in that theory the electromagnetic field can propagate through an euclidean vacuum in which there is no source present. In curved space-time on the other hand there is always a source present in the vacuum { 1 } and the source term is made up partly of a quaternion valued canonical electromagnetic energy momentum, T^{μ}.

In general relativity therefore the $\underline{B}^{(3)}$ field is quaternion valued, and represented by:

$$B^{(3)} = \frac{\Phi^{(0)} R}{8} \left(q_1 q_2^* - q_2 q_1^* \right) \quad - (16)$$

using a choice of metric so that the field is phase free. More generally the quaternion valued metric is represented by:

$$q^{\mu} = \left(q^{\mu(0)}, q^{\mu(1)}, q^{\mu(2)}, q^{\mu(3)} \right) \quad - (17)$$

and there is an infinite set of longitudinal field components in curved space-time, each component being determined by a particular metric and scalar curvature in eqn. (15). Therefore O(3) electrodynamics is a sub-theory of the general theory represented by eqn. (15), a sub-theory which is also developed in conformally curved space-time {13,14}.

On the other hand U(1) electrodynamics is developed in Euclidean space-time, and is an Abelian theory in which the electromagnetic field can propagate through a flat space-time vacuum which contains no charge or mass, and which can propagate without a source term, a non-sequitur.

SUMMARY

In this paper it has been shown that electrodynamics has in general a non-Abelian structure, represented in gauge field theory by O(3) electrodynamics { 11 }, which is able to reproduce phenomena inaccessible to U(1) electrodynamics. On the most general level electrodynamics in curved space-time must have a non-Abelian structure represented by eqn. (15) and so cannot be represented by a U(1) sector symmetry as in the received view. One of the important consequences of the non-Abelian structure electromagnetic theory is that there exists a source of energy in curved space-time, energy which can be used for practical purposes {9, 10}.

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http://www.ott.doe.gov/electromagnetic/

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