

# 163(4): General Theory of Elastic Scattering

This is a special case of note 160(3).

$$\gamma m_1 c^2 + m_2 c^2 = \gamma' m_1 c^2 + \gamma'' m_2 c^2 \quad - (1)$$

with

$$\gamma = \gamma' \quad - (2)$$

$$\omega = \omega' \quad - (3)$$

Here  $m_1$  is the mass of the incoming object, such as an electron or neutron, and  $m_2$  the mass of a static object such as an atom in a lattice. In elastic electron or neutron scattering the energy of the electron or neutron does not change, i.e. eqs. (2) and (3) are true. The atom is initially static, so its initial energy is  $m_2 c^2$ .

Eq. (23) of note 160(3) gives:

$$m_2 = \frac{\hbar \omega''}{c^2} \quad - (4)$$

$$\hbar \omega'' = m_2 c^2 \quad - (5)$$

i.e.

This means that the atom's energy after collision is its rest energy, i.e. the same as its initial energy.

So far the theory gives a sensible result. However, the rigorous (correct) consideration of momentum exchange leads to eq. (32) of note 160(3):

$$\omega''^2 \gamma''^2 = \omega^2 \gamma^2 + \omega'^2 \gamma'^2 - 2\omega\omega'\gamma\gamma'\cos\theta \quad - (6)$$

$$2) \text{ i.e. } K''^2 = K^2 + K'^2 - 2KK' \cos \theta. \quad - (7)$$

In elastic scattering:

$$K^2 = K'^2 \quad - (8)$$

$$\text{so } \omega^2 v^2 = \omega'^2 v'^2 \quad - (9)$$

$$\text{i.e. } \omega''^2 v''^2 = 2\omega^2 v^2 (1 - \cos \theta). \quad - (10)$$

Eq. (6) of note 160(4) then becomes:

$$\omega \omega' = x_1^2 + (\omega^2 - x_1^2)^{1/2} (\omega'^2 - x_1^2)^{1/2} \cos \theta \quad - (11)$$

$$\text{for all } x_2. \text{ Using eq. (3):}$$

$$\omega^2 = x_1^2 + (\omega^2 - x_1^2) \cos \theta \quad - (12)$$

$$\text{so either: } x_1 = \omega \quad - (13)$$

$$\text{or } \cos \theta = 1. \quad - (14)$$

Eq. (13) means, for all  $x_2$ :

$$m_1 c^2 = \hbar \omega \quad - (15)$$

which is a fundamental contradiction, because

$m_1$  is initial moving, so:

$$\gamma m_1 c^2 = \hbar \omega \quad - (16)$$

$$\gamma \neq 1.$$

Eq. (14) means no scattering, smarter contradiction.  
The theory of scattering fails catastrophically.