1) 164(1):

The Covariant Mass Ratio in Ninety Degree Scattering.

As in note 160(3), after ninety degree scattering of different masses, the result after collision is:

$$\omega' = \frac{x_2 \omega + x_1^2}{\omega + x_2} \quad -(1)$$

with

$$x_2 = \omega' + \omega'' - \omega \quad -(2)$$

Therefore:

$$x_1^2 = \omega'(\omega' + \omega'') - \omega(\omega' + \omega'' - \omega) \quad -(3)$$

Therefore after collision:

$$\hbar(\omega' + \omega'' - \omega) = m_2 c^2 \quad -(4)$$

$$\hbar\left[\omega'(\omega' + \omega'') - \omega(\omega' + \omega'' - \omega)\right]^{1/2} = m_1 c^2 \quad -(5)$$

$$= \hbar\left[(\omega' - \omega)(\omega' + \omega'') + \omega^2\right]^{1/2}$$

It is known that before collision

$$\hbar \omega_0 = m_2 c^2 \quad -(6)$$

$$\hbar \omega = m_1 c^2 \quad -(7)$$

It is seen that m_1 and m_2 are frequency dependent.
The rest masses are defined by:

$$m_2 = m_{20} \quad -(8)$$

$$m_1 = \gamma m_{10} \quad -(9)$$

2) Because m_2 is static initially.

Therefore, before collision, the covariant mass ratios are:

$$\frac{m_1}{m_{10}} = \gamma, \quad \frac{m_2}{m_{20}} = 1 \quad -(10)$$

and after collision:

$$\left(\frac{m_1}{m_{10}}\right)^2 = \frac{R_1}{R_{10}} = \gamma^2 \left(\frac{(\omega' - \omega)(\omega' + \omega'') + \omega^2}{\omega^2}\right) \quad -(11)$$

$$\left(\frac{m_2}{m_{20}}\right)^2 = \frac{R_2}{R_{20}} = \left(\frac{\omega' + \omega'' - \omega}{\omega_0}\right)^2 \quad -(12).$$

Eq. (11) is:

$$\boxed{\frac{R_1}{R_{10}} = \gamma^2 \left[1 + \frac{1}{\omega^2}(\omega' - \omega)(\omega' + \omega'')\right]} \quad -(13)$$

and eq. (12) is

$$\boxed{\frac{R_2}{R_{20}} = \left(\frac{\omega'}{\omega_0} + \frac{\omega''}{\omega_0} - \frac{\omega}{\omega_0}\right)^2} \quad -(14)$$

The de Broglie energy postulate is modified to:

$$mc^2 = \hbar\omega \quad -(15)$$

where m is the dynamical mass.