1)

164(3): Mass Ratio Spectrum for General Scattering.

From note 160(3):

$$x_2 = \frac{\omega\omega'}{\omega-\omega'} - \left(\frac{x_1^2 + (\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2}\cos\theta}{\omega-\omega'} \right) \quad -(1)$$

$$= \omega'' + \omega' - \omega$$

$$= \omega_0$$

Therefore:

$$x_1^2 + (\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2}\cos\theta = \omega\omega' - \omega_0(\omega - \omega')$$

$$= A \quad -(2)$$

i.e.

$$x_1^4(1-\cos^2\theta) + x_1^2\left((\omega^2+\omega'^2)\cos^2\theta - 2A\right) + A^2 - \omega^2\omega'^2\cos^2\theta = 0 \quad -(3)$$

so

$$x_1^2 = \frac{1}{2a}\left(-b \pm (b^2 - 4ac')^{1/2}\right) \quad -(4)$$

$$a = 1-\cos^2\theta,$$
$$b = (\omega'^2 + \omega^2)\cos^2\theta - 2A,$$
$$c' = A^2 - \omega^2\omega'^2\cos^2\theta,$$
$$A = \omega\omega' - \omega_0(\omega - \omega')$$

and

$$\boxed{\frac{R_1}{R_2} = \frac{x_1^2}{\omega_0^2}} \quad -(5)$$

The spectrum R_1/R_2 can be plotted

2) using the data of note 160(5):

$$\omega = 2.5067 \times 10^{18} \text{ rad s}^{-1} \quad -(6)$$

and

θ/rad	$\omega'/10^{18}$ (rad s^{-1})
0.6981	2.1967
0.8726	2.1222
1.0908	2.0355
1.3089	1.9528
1.4835	1.8908
1.74525	1.8081

This gives a two dimensional map of R_1/R_2 as a function of ω' and θ, as being given by eq. (1) and ω_0 by:

$$\omega_0 = m_2 c^2/\hbar \quad -(7)$$

$$m_2 = 1.99 \times 10^{-26} \text{ kg}, \quad -(8)$$

the mass of a carbon atom.