181(1): Relativistic Hamilton Jacobi Equation for R.

The effect of an electromagnetic potential A^μ on the Einstein energy equation results in the relativistic Hamilton Jacobi equation:

$$(p^\mu - eA^\mu)(p_\mu - eA_\mu) = m_0^2 c^2 \quad -(1)$$

This approach has the advantage of a well defined hamiltonian

$$H = \frac{1}{2} m_0 c^2 \quad -(2)$$

Here m_0 is the measured mass. Since this is such a familiar concept it is an advantage to develop a parallel approach based on the definition developed in UFT 180 and previous work:

$$R = \kappa^\mu \kappa_\mu = \frac{\omega^2}{c^2} - \kappa^2 \quad -(3)$$

The electromagnetic potential is related to κ^μ by:

$$p^\mu = eA^\mu = \hbar \kappa^\mu \quad -(4)$$

which is a combination of the minimal prescription and the basic postulate of quantum mechanics:

$$p^\mu = \hbar \kappa^\mu \quad -(5)$$

i.e.

$$E = \hbar\omega, \quad \underline{p} = \hbar \underline{\kappa} \quad -(6)$$

Therefore in eq. (1):

$$A^\mu = \frac{\hbar}{e} \kappa^\mu , \quad A_\mu = \frac{\hbar}{e} \kappa_\mu \quad -(7)$$

and in eq. (3):

$$R = \left(\frac{e}{\hbar}\right)^2 A^\mu A_\mu \quad -(8)$$

so R is the electromagnetic R. More generally, in ECE theory:

$$R = \left(\frac{e}{\hbar}\right)^2 A^\mu_{\ a} A^a_\mu \quad -(9)$$

where

$$A^a_\mu = A^{(0)} q^a_\mu . \quad -(10)$$

Using the tetrad normalization:

$$q^a_\mu q^\mu_a = 1 \quad -(11)$$

then

$$R = \left(\frac{e}{\hbar}\right)^2 A^{(0)2} \quad -(12)$$

The wave equation found in UFT 180 is:

$$(\Box + R)\phi = 0 \quad -(13)$$

where

$$R = \frac{\omega^2}{c^2} - \kappa^2 . \quad -(14)$$

In order to put this into the Hamilton Jacobi format R is divided as follows:

$$R = R_0 + R_1 \quad -(15)$$

3) where:

$$R_0 = \left(\frac{m_0 c}{\hbar}\right)^2 = \frac{\omega_0^2}{c^2} - \kappa_0^2 \quad -(16)$$

and

$$R_1 = \frac{\omega_1^2}{c^2} - \kappa_1^2, \quad -(17)$$

so:

$$\left(\Box + R_1 + \left(\frac{m_0 c}{\hbar}\right)^2\right)\phi = 0. \quad -(18)$$

in which

$$-\hbar^2 \Box = - p^\mu p_\mu. \quad -(19)$$

Therefore:

$$p^\mu p_\mu = \hbar^2 R_1 + m_0^2 c^2 \quad -(20)$$

or

$$\boxed{p^\mu p_\mu - \hbar^2 R_1 = m_0^2 c^2} \quad -(21)$$

This is a Hamilton Jacobi type structure, where

$$R_1 = \kappa_1^\mu \kappa_{1\mu}, \quad -(22)$$

where

$$\kappa_1^\mu = \left(\frac{\omega_1}{c}, \underline{\kappa_1}\right) \quad -(23)$$

In the case of electromagnetism:

$$\kappa_1^\mu = \frac{e}{\hbar} A_1^\mu \quad -(24)$$

so:

$$p^\mu p_\mu - e^2 A_1^\mu A^1_\mu = \left(\frac{m_0 c}{\hbar}\right)^2 \quad -(25)$$

Eq. (15) is equivalent to defining the covariant mass m by:

$$\boxed{m = m_0 + m_1} \quad -(26)$$

i.e. the effect of the interaction of a particle with an electromagnetic field is to change the measured mass m_0 by an amount m_1.

Eqns. such as (25) have the advantage of using the well defined hamiltonian (2), the hamiltonian used in metric based theory of general relativity.

If there is no e/m field present, eq. (25) becomes the Einstein energy equation:

$$p^\mu p_\mu = m_0^2 c^2 \quad -(27)$$

Finally, eq. (1) "factorizes" into the first order differential fermion equation. This will be the subject of the next note

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