1) 190(7): Testing the Standard Model for the Precessing Ellipse.

The equation of the precessing ellipse is:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(1)$$

where x is a constant.

Therefore:

$$\frac{dr}{d\theta} = \frac{x\alpha\epsilon\sin(x\theta)}{(1+\epsilon\cos(x\theta))^2} \quad -(2)$$

So:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{x^2\alpha^2\epsilon^2(1-\cos^2(x\theta))}{(1+\epsilon\cos(x\theta))^4} \quad -(3)$$

$$= r^4\left(\frac{1}{b^2} - m(r)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)$$

i.e.

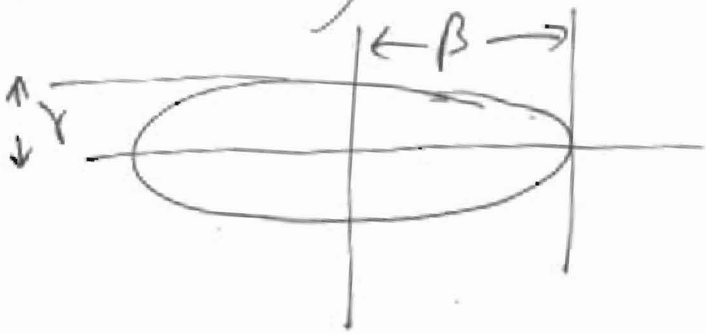
$$m(r) = \frac{a^2}{r^2(a^2+r^2)}\left[\frac{r^4}{b^2} - \left(\frac{x\alpha\epsilon}{(1+\epsilon\cos(x\theta))^2}\right)^2\left(1-\cos^2(x\theta)\right)\right] \quad -(4)$$

where

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r}-1\right) \quad -(5)$$

$$\epsilon = \left(1-\left(\frac{\beta}{\gamma}\right)^2\right)^{1/2} \quad -(6)$$

$$\alpha = \frac{\beta^3}{\gamma^2} \quad -(7)$$



2)

Therefore α and ϵ can be found by astronomical observations of the aphelion and perihelion. In the solar system the precession is small but in binary pulsars it is a large effect. The constants a and b are:

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E} \quad -(8)$$

so

$$b = \left(\frac{mc^2}{E}\right) a. \quad -(9)$$

The parameters a and x can be regarded as fitting parameters.

According to the standard model:

$$m(r) = 1 - \frac{2MG}{c^2 r} \quad -(10)$$

<u>Computational Problem</u>

Can the function (4) be reproduced by the function (10) for observed ϵ and α? It is possible to model the problem by using any β and γ as input, and varying a and x, or fitting for a and x.

By investigating orbits that are very close to being circular, such as the

3) Earth's orbit, then:

$$\epsilon \to 0 \qquad -(11)$$

and from eq. (4):

$$m(r) \to \left(\frac{a}{b}\right)^2 \left(\frac{r^2}{a^2 + r^2}\right)$$

$$= \left(\frac{E}{mc^2}\right)^2 \left(1 + \left(\frac{a}{r}\right)^2\right)^{-1} \qquad -(12)$$

where

$$a = \frac{L}{mc} \qquad -(13)$$

So:

$$\boxed{m(r) \to \left(\frac{E}{mc^2}\right)^2 \left(1 + \left(\frac{L}{mcr}\right)^2\right)^{-1}} \qquad -(14)$$

for a circular orbit.

If:

$$L << mcr \qquad -(15)$$

then

$$m(r) \sim \left(\frac{E}{mc^2}\right)^2 \left(1 - \left(\frac{L}{mcr}\right)^2\right) \qquad -(16)$$

and this is not eq. (10).

So it seems that the standard model fails.

4)

For a circular orbit:

$$L = mvr \quad -(17)$$

where v is the orbital linear velocity. So:

$$m(r) \sim \left(\frac{E}{mc^2}\right)\left(1 - \frac{v^2}{c^2}\right) \quad -(18)$$

$$= \frac{E}{\gamma^2 mc^2} \quad -(19)$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(20)$$

The total energy is:

$$E = m(r)\, mc^2 \frac{dt}{d\tau} \quad -(21)$$

and the angular momentum is

$$L = mr^2 \frac{d\theta}{d\tau}. \quad -(22)$$

The infinitesimal line element is:

$$ds^2 = c^2 d\tau^2 = m(r)\, c^2 dt^2 - m^{-1}(r)\, dr^2 - r^2 d\theta^2 \quad -(23)$$

By definition:

$$\underline{dr} \cdot \underline{dr} = m^{-1}(r)\, dr^2 - r^2 d\theta^2$$

$$= v^2 dt^2 \quad -(24)$$

In the frame in which m is at rest:

5)

$$v = 0 \quad -(25)$$

So $$d\tau^2 = m(r)\,dt^2 \quad -(26)$$

i.e. $$\frac{dt}{d\tau} = \left(m(r)\right)^{-1/2} \quad \text{if } v = 0. \quad -(27)$$

Otherwise: $$d\tau^2 = m(r)\,dt^2 - \frac{v^2}{c^2}\,dt^2$$

$$= \left(m(r) - \frac{v^2}{c^2}\right) dt^2 \quad -(28)$$

and $$\boxed{\frac{dt}{d\tau} = \left(m(r) - \frac{v^2}{c^2}\right)^{-1/2}} \quad -(29)$$

So $$\boxed{\begin{aligned} E &= mc^2\, m(r) \left(m(r) - \frac{v^2}{c^2}\right)^{-1/2} \\ &= \text{constant} \end{aligned}} \quad -(30)$$

$$\boxed{\begin{aligned} L &= mr^2 \frac{d\theta}{dt}\frac{dt}{d\tau} \\ &= mr^2 \omega \left(m(r) - \frac{v^2}{c^2}\right)^{-1/2} \\ &= \text{constant} \end{aligned}} \quad -(31)$$