

1) 190(10): Correct Newtonian Limits

Orbits

The orbit can be written as:

$$\frac{dr}{d\theta} = r^2 \left( \frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{1/2} \quad - (1)$$
$$= r^2 \left( \frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2} + \frac{r_0}{a^2 r} + \frac{r_0}{r^3} \right)^{1/2}$$

As  $r \rightarrow \infty$  — (2)

$$\frac{dr}{d\theta} = r^2 \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \quad - (3)$$

This is a spiral, not a precessing ellipse.

Areal Velocity (Kepler's Second Law).

$$\frac{dA}{dt} = \frac{1}{2} \frac{c^2 L}{E} \left(1 - \frac{r_0}{r}\right) \rightarrow \frac{1}{2} \frac{c^2 L}{E} \quad - (4)$$

= constant.

Orbital Linear Velocity:

$$v = cb \left(1 - \frac{r_0}{r}\right) \left( \frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{1/2} \quad - (5)$$

As  $r \rightarrow \infty$

$$v \rightarrow cb \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \quad - (6)$$

= constant

i.e. the orbit goes to a circle.