

Note 230(4)a : Check of Eq. (5)

This equation is obtained by defining:

$$\begin{aligned} (g_{\mu}^a p^{\mu}) p_{\mu} &:= g_{\mu}^a (p^{\mu} p_{\mu}) \\ &= m^2 c^2 g_{\mu}^a \end{aligned} \quad - (1)$$

This implies:

$$\begin{aligned} g_{\mu}^a &= \frac{1}{m^2 c^2} p^a p_{\mu} \\ &= \frac{1}{m^2 c^2} (g_{\mu}^a p^{\mu}) p_{\mu} \\ &= p^a p_{\mu} / (m^2 c^2) \end{aligned} \quad - (2)$$

$$\begin{aligned} \text{So: } p^a &= g_{\mu}^a g^{\mu} = \frac{p^a p_{\mu}}{m^2 c^2} p^{\mu} \\ &= p^a \left( p_{\mu} p^{\mu} / (m^2 c^2) \right) \\ &= p^a \end{aligned} \quad \text{QED.}$$

So  $p^a p_{\mu}$  is a tensor product.

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