

334(2) : The Anomalous Zeeman Effect is of Type One Hamiltonian

In addition to the description of the Zeeman effect is note 334(1) there is an additional spin term which appears on quantization in the $SU(2)$ basis:

$$\begin{aligned} H\psi &= \frac{1}{m} \underline{\sigma} \cdot \left(-i\hbar \underline{\nabla} \left(\frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot (\underline{p}_0 - e\underline{A}) \psi \right) \right) \\ &\quad - \frac{e}{m} \underline{\sigma} \cdot \underline{A} \left(\frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot (\underline{p}_0 - e\underline{A}) \psi \right) \\ &= \frac{ie\hbar}{m} \underline{\sigma} \cdot \underline{\nabla} \left(\frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{A} \psi \right) + \dots + \dots \\ &= \frac{ie\hbar}{m} \left(\frac{\gamma^2}{1+\gamma} \right) \underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{A} \psi + \dots \quad - (1) \end{aligned}$$

$$\text{so } \text{Re } H\psi = - \frac{e\hbar}{m} \left(\frac{\gamma^2}{1+\gamma} \right) \underline{\sigma} \cdot \underline{B} \quad - (2)$$

$$\text{where } \underline{B} = \underline{\nabla} \times \underline{A} \quad - (3)$$

$$\text{Defining: } \underline{S} = \frac{\hbar}{2} \underline{\sigma} \quad - (4)$$

The complete Zeeman effect Hamiltonian of last two is:

$$H = - \frac{e}{m} \left(\frac{\gamma^2}{1+\gamma} \right) (\underline{L} + 2\underline{S}) \cdot \underline{B} \quad - (5)$$

This gives the usual Hamiltonian of Dirac if

$$\frac{\gamma^2}{1+\gamma} \rightarrow \frac{1}{2} \quad - (6)$$

2) so the usual Dirac approximation is equivalent to:

$$V \rightarrow \frac{1}{2} - (7)$$

or

$$V \rightarrow \frac{1}{2} - (8)$$

as Dirac pointed out in his 1933 Nobel lecture.

It is well known that eq. (5) can be expressed

as:

$$H = - \frac{e}{m} \left(\frac{V^2}{1+V} \right) g_J \underline{J} \cdot \underline{B} - (9)$$

where the Landé factor is:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} - (10)$$

The J quantum number, referred by Sommerfeld, is:

$$J = L + S, \dots, L - S - (11)$$

with:

$$J_z \psi = m_J \hbar \psi - (12)$$

$$m_J = -J, \dots, J - (13)$$

and

$$J^2 \psi = J(J+1) \psi - (14)$$

Therefore:

$$H \psi = - \frac{e \hbar}{m} \left(\frac{V^2}{1+V} \right) g_J m_J B_z \psi - (15)$$

and

$$E = \langle H \rangle = - \frac{e \hbar}{m} \left(\frac{V^2}{1+V} \right) g_J m_J B_z - (16)$$

3) The energy levels of H atom are therefore:

$$E_H = -\frac{1}{2} mc^2 \left(\frac{d}{n} \right)^2 - \frac{e\hbar}{m} \left(\frac{\gamma^2}{1+\gamma} \right) g_J m_J \hbar_Z - (17)$$

where $m_J = -J, \dots, J - (18)$

As in Note 334(1), $\gamma^2/(1+\gamma)$ can be developed either as a function:

$$\frac{\gamma^2}{1+\gamma} = \left(1 - \frac{p_0^2}{m^2 c^2} + \left(1 - \frac{p_0^2}{m^2 c^2} \right)^{1/2} \right)^{-1} - (19)$$

or as an expectation value:

$$\frac{\gamma^2}{1+\gamma} = \left(1 - \left(\frac{d}{n} \right)^2 + \left(1 - \left(\frac{d}{n} \right)^2 \right)^{1/2} \right)^{-1} - (20)$$

where d is the fine structure constant:

$$d = 7.297351 \times 10^{-3} - (21)$$

The selection rules are:

$$\Delta J = 0, \pm 1 - (22)$$

$$J=0 \rightarrow J=0 - (23)$$

$$\Delta m_J = 0, \pm 1, - (24)$$

and

$$\Delta n = \text{any} - (25)$$