

DEVELOPMENT OF ECE2: GRAVITATIONAL FIELD EQUATIONS,  
ANTISYMMETRY, EQUIVALENCE PRINCIPLES, COUNTER GRAVITATION  
AND AHARONOV BOHM EFFECTS.

by

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ABSTRACT

The gravitational field / potential equations are derived of ECE2 theory and applied to counter gravitation. It is shown that the gravitational force between  $m$  and  $M$  can vanish under well defined conditions, and can become positive, so  $m$  repels  $M$ , and a craft may be lifted off the ground by an on board electric device. The antisymmetry laws of ECE2 are derived and used to derive the Newtonian equivalence principle from geometry. The theory of spin connection resonance is developed and shown to result in zero gravitation. The Aharonov Bohm effects of vacuum ECE2 theory are developed.

Keywords: Development of ECE2 theory, gravitational field / potential equations, counter gravitation, antisymmetry laws, derivation of the equivalence principle, vacuum ECE2 theory.

UFT 318



## 1. INTRODUCTION

In recent papers of this series {1 - 12} the ECE2 theory has been developed and applied to derive the field / potential equations of electrodynamics. In Section 2, ECE2 is applied to find the field / potential equations of gravitation. It is shown that the latter can produce zero or repulsive gravitation under well defined conditions, and that counter gravitation can be designed with an on board electrical device. The antisymmetry laws of ECE2 are derived and shown to result immediately in the Newtonian equivalence principle, thus deriving it from geometry. The EC2 theory allows potentials to exist in the absence of fields, Aharonov Bohm effects which are used to define the vacuum ECE2 theory needed to understand the principles of energy from spacetime.

This paper should be read with its background notes posted with UFT318 on [www.aias.us](http://www.aias.us). Note 318(1) derives the gravitational field / potential equations from geometry and the methods of UFT313 to UFT317. Note 318(2) derives the antisymmetry principles from which the Newtonian equivalence principle follows directly. Note 318(3) gives the ECE2 theory of spin connection resonance, which is applied to counter gravitation in Note 318(4). The ECE2 theory is applied to the Aharonov Bohm effects in Note 318(5), giving the basics of the vacuum ECE2 theory. Notes 318(6) and 318(7) develop the theory of zero gravitational force between  $m$  and  $M$ , and repulsion between  $m$  and  $M$ .

Section 3 is a numerical and graphical analysis of the main results of Section 2.

## 2. DEVELOPMENT OF ECE2

Following the principles and methods developed in the immediately preceding papers UFT313 to UFT317 it is possible to infer the gravitational field equations of ECE2 theory, which have the same structure as the electromagnetic field equations. The four

gravitational field equations of ECE2 theory are as follows:

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad - (1)$$

$$\underline{\nabla} \times \underline{g} + \frac{d\underline{\Omega}}{dt} = - (c\underline{\kappa}_0 \underline{\Omega} + \underline{\kappa} \times \underline{g}) = \frac{4\pi G}{c} \underline{J}_\Omega \quad - (2)$$

$$\underline{\nabla} \cdot \underline{\Omega} = \underline{\kappa} \cdot \underline{\Omega} = \frac{4\pi G}{c} \rho_\Omega \quad - (3)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \frac{d\underline{g}}{dt} = \frac{\underline{\kappa}_0}{c} \underline{g} + \underline{\kappa} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m \quad - (4)$$

Here  $\underline{g}$  is the gravitational field,  $G$  is Newton's constant,  $\rho_m$  is the mass density,  $\underline{J}_m$  is the current of mass density,  $\underline{\Omega}$  is the gravitomagnetic field,  $\rho_\Omega$  is the gravitomagnetic mass density, and  $\underline{J}_\Omega$  is the current of gravitomagnetic mass density. In these equations:

$$\underline{\kappa}_0 = 2 \left( \frac{\underline{v}_0}{r^{(0)}} - \underline{\omega}_0 \right) \quad - (5)$$

and

$$\underline{\kappa} = 2 \left( \frac{\underline{v}}{r^{(0)}} - \underline{\omega} \right) \quad - (6)$$

where the tetrad and spin connection four vectors are respectively

$$\underline{v}_\mu = (\underline{v}_0, -\underline{v}) \quad - (7)$$

and

$$\underline{\omega}_\mu = (\omega_0, -\underline{\omega}) \quad - (8)$$

The field potential relations are

$$\underline{g} = -\underline{\nabla} \underline{\Phi} - \frac{\partial \underline{Q}}{\partial t} + 2 \left( c \underline{\omega}_0 \underline{Q} - \underline{\Phi} \underline{\omega} \right) \quad - (9)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} + 2 \underline{\omega} \times \underline{Q} \quad - (10)$$

where the mass / current density four vector is:

$$\underline{J}_m^\mu = (c \rho_m, \underline{J}_m) \quad - (11)$$

and where the gravitational vector four potential is:

$$\underline{Q}^\mu = \left( \underline{\Phi}, c \underline{Q} \right) \quad - (12)$$

In electrodynamics it is almost always assumed that the magnetic charge current density is zero. The parallel assumption in ECE2 gravitational theory leads to a simpler set of equations:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad - (13)$$

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad - (14)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad - (15)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \frac{\partial \underline{g}}{\partial t} = \underline{\kappa} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m \quad - (16)$$

whose structure is superficially similar to the Maxwell Heaviside (MH) equations of electrodynamics. However, Eqs. ( 13 ) to ( 16 ) are equations of a generally covariant unified field theory (ECE2 theory).

From Eqs. ( 15 ) and ( 16 ):

$$\frac{4\pi G}{c^2} \underline{\nabla} \cdot \underline{J}_m = -\frac{1}{c^2} \frac{d}{dt} (\underline{\nabla} \cdot \underline{g}) = -\frac{4\pi G}{c^2} \frac{d\rho_m}{dt} \quad - (17)$$

so it follows that

$$\frac{d\rho_m}{dt} + \underline{\nabla} \cdot \underline{J}_m = 0 \quad - (18)$$

i.e.

$$d_\mu J_m^\mu = 0 \quad - (19)$$

which is the equation of conservation of mass current density in ECE2 theory.

In the Newtonian universal gravitation, it is known experimentally that:

$$\underline{g} = g_r \underline{e}_r = -\frac{MG}{r^2} \underline{e}_r \quad - (20)$$

so it follows that:

$$\frac{dg_r}{dr} = \frac{2MG}{r^3} = 2g_r \left( \frac{1}{r^{(2)}} \underline{v}_r - \omega_r \right) = -\frac{2MG}{r^2} \left( \frac{1}{r^{(2)}} \underline{v}_r - \omega_r \right) \quad - (21)$$

and

$$\kappa_r = -\frac{2}{r} \quad - (22)$$

Eq. ( 22 ) shows how ECE2 reduces to the Newtonian theory. In the latter theory, the only field equations used are:

$$\underline{g} = -\underline{\nabla} \Phi - (23)$$

and:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m - (24)$$

together with the Newtonian equivalence principle:

$$\underline{F} = m \underline{g} = -\frac{mMG}{r^2} \underline{e}_r - (25)$$

Eq. (25) in the Newtonian theory is a theoretically unproven assumption. It happens to produce the experimental data. In ECE2 however Eq. (25) can be derived from geometry and antisymmetry as follows.

Consider the ECE2 generalizations of the Coulomb and Newton laws:

$$\underline{\nabla} \cdot \underline{E} = \underline{\kappa} \cdot \underline{E} = \frac{\rho_e}{\epsilon_0} - (26)$$

and

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m - (27)$$

where  $\rho_e$  is the electric charge density,  $\underline{E}$  the electric field strength in volts per metre, and  $\epsilon_0$  the vacuum S. I. permittivity. The electric field strength is defined by:

$$\underline{E} = -\underline{\nabla} \phi_e - \frac{\partial \underline{A}}{\partial t} + 2(c\omega_0 \underline{A} - \phi_e \underline{\omega}) - (28)$$

where the electromagnetic four potential is:

$$A^\mu = (\phi_e, \underline{\Sigma}_e) - (29)$$

The antisymmetry laws {1 - 12} of ECE2 are therefore

$$-\underline{\nabla} \underline{\phi}_e + 2c\omega_0 \underline{A} = -\frac{\partial \underline{A}}{\partial t} - 2\underline{\phi}_e \underline{\omega} \quad (30)$$

and

$$-\underline{\nabla} \underline{\Phi}_m + 2c\omega_0 \underline{Q} = -\frac{\partial \underline{Q}}{\partial t} - 2\underline{\Phi}_m \underline{\omega} \quad (31)$$

In the absence of a vector potential  $\underline{A}$  and gravitomagnetic potential  $\underline{Q}$  :

$$\underline{E} = -\underline{\nabla} \underline{\phi}_e = -2\underline{\phi}_e \underline{\omega} \quad (32)$$

and

$$\underline{g} = -\underline{\nabla} \underline{\Phi}_m = -2\underline{\Phi}_m \underline{\omega} \quad (33)$$

The Newtonian equivalence principle ( 25 ) follows immediately from Eq. ( 33 ) :

$$\underline{F} = m\underline{g} = -m\underline{\nabla} \underline{\Phi} = -2m\underline{\Phi} \underline{\omega} \quad (34)$$

where the gravitational potential of Newtonian universal gravitation is:

$$\underline{\Phi} = -\frac{M\underline{G}}{r} \quad (35)$$

Here  $M$  is the mass of an object that attracts the mass  $m$ . So:

$$\underline{F} = m\underline{g} = -\frac{mM}{r^2} \underline{e}_r = -\frac{2mM\underline{G}}{r} \underline{\omega} \quad (36)$$

It follows that the spin connection vector is:

$$\underline{\omega} = \frac{i}{2r} \underline{e}_r. \quad - (37)$$

Similarly in electrostatics:

$$\underline{F} = e \underline{E} = -e \underline{\nabla} \phi_e = -2m \phi_e \underline{\omega} \quad - (38)$$

where:

$$\phi_e = -\frac{e_1}{4\pi \epsilon_0 r} \quad - (39)$$

so:

$$\underline{F} = e \underline{E} = -\frac{e e_1}{4\pi \epsilon_0 r^2} \underline{e}_r = -\frac{2 e_1}{4\pi \epsilon_0 r} \underline{\omega} \quad - (40)$$

and the spin connection vector is again:

$$\underline{\omega} = \frac{1}{2r} \underline{e}_r. \quad - (41)$$

In the absence of a vector potential:

$$\underline{E} = -\underline{\nabla} \phi_e - 2 \phi_e \underline{\omega} \quad - (42)$$

and:

$$\underline{\nabla} \cdot \underline{E} = \rho_e / \epsilon_0 \quad - (43)$$

so:

$$(\nabla^2 + k_0^2) \phi_e = -\rho_e / \epsilon_0 \quad - (44)$$

where  $k_0^2$  is defined as:

$$k_0^2 = 2\bar{\nabla} \cdot \underline{\omega} \quad - (45)$$

Eq. (44) becomes an undamped Euler Bernoulli equation with the following choice of electric charge density:

$$\frac{\rho_e}{\epsilon_0} = -A \cos(kz) \quad - (46)$$

so the Euler Bernoulli equation is:

$$\frac{\partial^2 \phi_e}{\partial z^2} + k_0^2 \phi_e = A \cos(kz) \quad - (47)$$

whose solution is:

$$\phi_e = \frac{A \cos(kz)}{k_0^2 - k^2} \quad - (48)$$

Similarly in gravitational theory:

$$(\nabla^2 + k_0^2) \underline{\Phi}_m = -4\pi G \rho_m \quad - (49)$$

This is an Euler Bernoulli equation if the following definition is made:

$$4\pi G \rho_m = -A \cos(kz) \quad - (50)$$

giving the solution

$$\underline{\Phi}_m = \frac{A \cos(kz)}{k_0^2 - k^2} \quad - (51)$$

Eq. (49) reduces to the Poisson equation of Newtonian dynamics:

$$\nabla^2 \underline{\Phi}_m = -4\pi G \rho_m \quad - (52)$$

when

$$k_0^2 = 2 \underline{\nabla} \cdot \underline{\omega} = 0 \quad - (53)$$

The acceleration due to the gravity of a laboratory mass  $m$  in ECE2 theory is:

$$\underline{g}_m = -\underline{\nabla} \underline{\Phi}_m - 2 \underline{\omega} \underline{\Phi}_m \quad - (54)$$

and the gravitational force between  $m$  and the mass of the earth,  $M$ , is:

$$\underline{F} = M \underline{g}_m \quad - (55)$$

In the  $Z$  axis:

$$\underline{\Phi}_m = \frac{A \cos(k_2 Z)}{k_0^2 - k_2^2} \quad - (56)$$

and:

$$-\underline{\nabla} \underline{\Phi}_m = -\frac{\partial \underline{\Phi}_m}{\partial Z} = \frac{A k_2 \sin(k_2 Z)}{k_0^2 - k_2^2} \quad - (56)$$

so:

$$g_z = \frac{A}{k_0^2 - k_2^2} \left( k_2 \sin(k_2 Z) - 2 \omega_z \cos(k_2 Z) \right) \quad - (57)$$

Under the condition:

$$\tan(k_2 Z) = \frac{2 \omega_z}{k_2} \quad - (58)$$

it follows that:

$$g_z = 0 \quad - (59)$$

and

$$F_z = \underline{M} g_z = 0 \quad - (60)$$

In this condition there is no gravitational attraction between m and M.

The gravitational potential energy in joules of the mass m is:

$$U_m = m \Phi_m \quad - (61)$$

and the electrostatic potential energy of a charge e is:

$$U_e = e \phi_e \quad - (62)$$

Therefore:

$$(\nabla^2 + k_0^2) U_m = -4\pi m \rho_m \quad - (63)$$

and

$$(\nabla^2 + k_0^2) U_e = -\frac{e\rho_e}{\epsilon_0} \quad - (64)$$

All forms of energy are interconvertible so:

$$(\nabla^2 + k_0^2) (U_m + U_e) = -\left(4\pi m \rho_m + \frac{e\rho_e}{\epsilon_0}\right) \quad - (65)$$

For a mass m of a kilogram and a charge e of a coulomb in a volume of one cubic metre:

$$\frac{e\rho_e}{\epsilon_0} \gg 4\pi m \rho_m \quad - (66)$$

and to an excellent approximation:

$$(\nabla^2 + k_0^2) (m \Phi_m + e \phi_e) = -\frac{e\rho_e}{\epsilon_0} \quad - (67)$$

This equation shows that gravitation can be engineered by an on board device that

produces the driving force:

$$A \cos(k_z z) = -\frac{e\rho_e}{\epsilon_0} \quad (68)$$

giving the Euler Bernoulli equation:

$$(\nabla^2 + k_0^2)(m\bar{\Phi}_m + e\phi_e) = A \cos(k_z z) \quad (69)$$

whose solution is:

$$m\bar{\Phi}_m + e\phi_e = \frac{A \cos(k_z z)}{k_0^2 - k_z^2} \quad (70)$$

In the Z axis:

$$\bar{\Phi}_{mz} = \frac{1}{m} \left( \frac{A \cos(k_z z)}{k_0^2 - k_z^2} - e\phi_e \right) \quad (71)$$

and so:

$$-\frac{\partial \bar{\Phi}_{mz}}{\partial z} = \frac{1}{m} \left[ \frac{A k_z \sin(k_z z)}{k_0^2 - k_z^2} - e \frac{\partial \phi_e}{\partial z} \right] \quad (72)$$

giving the acceleration due to gravity:

$$g_z = \frac{1}{m} \left[ \frac{A k_z \sin(k_z z)}{k_0^2 - k_z^2} - 2\omega_z \cos(k_z z) + e \left( \frac{\partial \phi_e}{\partial z} + 2 \frac{\omega_z}{k_z} \phi_e \right) \right] \quad (73)$$

There is no gravitational force between m and M under the conditions

$$\tan(k_z z) = \frac{2\omega_z}{k_z} \quad (74)$$

and

$$\frac{\partial \phi_e}{\partial z} = -2\omega_z \phi_e. \quad (75)$$

Numerical solutions of these equations are given in Section 3.

From Eqs. ( 71 ) and ( 68 ) the gravitational potential is:

$$\Phi_m = \frac{e}{m} \left[ \frac{\rho_e}{\epsilon_0 (k_z^2 - k_0^2)} - \phi_e \right]. \quad (76)$$

There is no gravitational force between m and M if  $\Phi_m$  is zero, so in this case:

$$\phi_e = \frac{\rho_e}{\epsilon_0 (k_z^2 - k_0^2)} \quad (77)$$

and the electric field strength needed for condition ( 77 ) is:

$$\underline{E} = -\underline{\nabla} \phi_e - 2\omega_z \phi_e. \quad (78)$$

When the electric field strength of an on board device contained within a vehicle of mass m is tuned to condition ( 77 ), the g forces between m and M vanish. The vehicle is no longer attracted to the earth's mass M. Finally the condition for a positive g is a negative spin connection  $\underline{\omega}$ , so using:

$$\underline{g} = -\underline{\nabla} \Phi_m + 2\underline{\omega} \Phi_m \quad (79)$$

it becomes positive, or repulsive, when

$$2\underline{\omega} \Phi_m > \underline{\nabla} \Phi_m \quad (80)$$

In this condition the mass m lifts off the ground.

Finally in this section, the ECE2 vacuum theory is developed briefly with the equations:

$$\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} + 2(\underline{\omega}_0 \underline{A} - \phi \underline{\omega}) = \underline{0} \quad - (81)$$

and

$$\underline{B} = \underline{\nabla} \times \underline{A} + 2\underline{\omega} \times \underline{A} = \underline{0} \quad - (82)$$

which show immediately that  $\phi$  and  $\underline{A}$  are non-zero when  $\underline{E}$  and  $\underline{B}$  are zero. These are the conditions for the well known Aharonov Bohm effects, potentials are observed experimentally to exist in the absence of fields. These are the ECE2 vacuum potentials describing the electromagnetic energy present in spacetime (the ECE2 vacuum). By antisymmetry:

$$-\underline{\nabla} \phi + 2\underline{\omega}_0 \underline{A} = -\frac{\partial \underline{A}}{\partial t} - 2\phi \underline{\omega} \quad - (83)$$

and it follows that the vacuum potentials are defined by:

$$-\underline{\nabla} \phi + 2\underline{\omega}_0 \underline{A} = \underline{0} \quad - (84)$$

$$-\frac{\partial \underline{A}}{\partial t} - 2\phi \underline{\omega} = \underline{0} \quad - (85)$$

$$\underline{\nabla} \times \underline{A} + 2\underline{\omega} \times \underline{A} = \underline{0} \quad - (86)$$

If it is assumed for the sake of simplicity that:

$$\underline{\omega}_0 = \underline{0} \quad - (87)$$

there are three equations in three unknowns:

$$-\underline{\nabla} \phi + 2\underline{\omega} \times \underline{A} = \underline{0} \quad - (88)$$

$$-\frac{\partial \underline{A}}{\partial t} - 2\phi \underline{\omega} = \underline{0} \quad - (89)$$

$$\underline{\nabla} \times \underline{A} + 2\underline{\omega} \times \underline{A} = \underline{0} \quad - (90)$$

and these can be solved numerically, giving  $\phi$ ,  $\underline{A}$ , and  $\underline{\omega}$ .

The energy-momentum contained in the vacuum is:

$$\underline{E}^{\mu} = \left( \frac{E}{c}, \underline{p} \right) = e \underline{A}^{\mu} = e \left( \frac{\phi}{c}, \underline{A} \right) \quad (91)$$

The ECE2 vacuum can be thought of as being made up of photons with energy momentum:

$$\underline{E}^{\mu} = e \underline{A}^{\mu} = \hbar \underline{\kappa}^{\mu} = \hbar \left( \frac{\omega}{c}, \underline{\kappa} \right) \quad (92)$$

The Einstein / de Broglie equations of the ECE2 vacuum are:

$$E = e\phi = \hbar\omega = \gamma mc^2 \quad (93)$$

and

$$\underline{p} = e\underline{A} = \hbar \underline{\kappa} = \gamma m \underline{v} \quad (94)$$

where  $m$  is the mass of the photon and where the Lorentz factor is:

$$\gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \quad (95)$$

The ECE fermion equation {1 - 12} can be used to describe the interaction of the ECE2 photons with a circuit modelled by one electron. In UFT311 the circuit design needed to take energy from the ECE vacuum was described in all detail and excellent agreement found between ECE and data.

### 3. NUMERICAL AND GRAPHICAL ANALYSIS

(Section by co authors Horst Eckardt and Douglas Lindstrom)

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# Development of ECE2: gravitational field equations, antisymmetry, equivalence principles, counter gravitation and Aharonov Bohm effects

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## 3 Numerical and graphical analysis

The conditions that no gravitational force between masses  $m$  and  $M$  should exist are given by Eqs.(74) and (75):

$$\tan(k_Z Z) = 2 \frac{\omega_Z}{k_Z}, \quad (96)$$

$$\frac{\partial \phi_e}{\partial Z} = -2\omega_Z \phi_e. \quad (97)$$

From Eq.(97) a solution for  $\phi_e$  can be determined which has to be inserted in Eq.(77) to give a charge density

$$\rho_e = \phi_e(k_Z^2 - k_0^2). \quad (98)$$

A consistency problem seems to remain because  $\rho_e$  was assumed to be strictly periodic by Eq.(46) which cannot be guaranteed by this approach, but we will see that the above conditions need only to be fulfilled at a certain point  $Z$  which is defined by  $k_Z$  to let the gravitational force vanish.

In Eqs.(96-97) the spin connection  $\omega_Z$  appears for which we have to introduce a model. For obtaining an Euler-Bernoulli equation,  $k_0$  has to be a constant. This means, that the divergence of  $\omega_z$  has to be constant as follows from Eq.(45). Therefore we make the first approach

$$\omega_Z = \frac{Z}{Z_0^2} \quad (99)$$

with a new constant  $Z_0$ . The squared constant in the denominator is required to obtain the right physical dimension  $1/m$ . Eq.(97) then has the solution

$$\phi_e(Z) = \phi_0 e^{-\frac{Z^2}{Z_0^2}} \quad (100)$$

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with a constant  $\phi_0$ . From (96) follows

$$\tan(k_Z Z) = \frac{2\omega_Z}{k_Z} = \frac{2Z}{k_Z Z_0^2}. \quad (101)$$

This is a transcendental equation. Approximating the tangens function by the first two terms of its taylor expansion around zero:

$$\tan(k_Z Z) \approx k_0 Z + \frac{k_0^3 Z^3}{3} \quad (102)$$

leads to an equation of third order which has the only non-trivial, positive solution

$$Z = \frac{\sqrt{3} \sqrt{2 - k_Z^2 Z_0^2}}{k_Z^2 Z_0}. \quad (103)$$

This means that the gravitational force can only be suppressed at this value of the  $Z$  coordinate. The potential  $\phi_e$  has to be tuned to have the suitable value for this  $Z$ . Both depend on the wave number  $k_Z$  which can be chosen freely in principle. The dependence of this  $Z$  value on  $k_Z$  is graphed in Fig. 1 for parameters  $Z_0 = 1$  m,  $\phi_0 = 100$  V. It is important to note that  $Z$  is defined only below a maximum value of  $k_Z$ . The dependence of  $\phi_e$  on  $k_Z$  is shown in Fig. 2. There is an onset at a minimal wave number. For the method to work, both  $Z(k_Z)$  and  $\phi_e(k_Z)$  need an overlapping  $k_Z$  range which is quite small in this case at about 1.2/m. The curves additionally depend on  $Z_0$ .

Another choice for  $\omega_Z$  could be

$$\omega_Z = \frac{1}{2Z} \quad (104)$$

which is valid for the Coulomb potential. Although this does not fulfill condition (45) for constancy, we used this to show the effect. Maybe one can restrict to a small portion of  $Z$  where it is nearly constant. We furthermore use cartesian coordinates. Then the solution for the electrical potential is

$$\phi_e(Z) = \frac{A_0}{Z} \quad (105)$$

with a constant  $A_0$ , i.e. a Coulomb-like potential. Eq.(96) gives a quartic equation in  $Z$  with the only real-valued, non-negative solution

$$Z = \frac{\sqrt{\sqrt{21} - 3}}{\sqrt{2} k_Z} = \frac{0.88954361752413}{k_Z}. \quad (106)$$

The resulting functions  $Z(k_Z)$  and  $\phi_e(k_Z)$  are graphed in Figs. 3 and 4. There is a broad overlapping range of  $k_Z$  now.

As the third and last approach we derive  $\omega_Z$  directly from Eq.(96):

$$\omega_Z(Z) = \frac{1}{2} k_Z \tan(k_Z Z). \quad (107)$$

Inserting this into (97) gives the astonishingly simple solution

$$\phi_e(Z) = \phi_0 \cos(k_Z Z). \quad (108)$$

By Eq.(98) this gives the original oscillating charge density  $\rho_e(Z)$ . The tangens function is linear near to  $Z = 0$ , so condition (45) is fulfilled too. This seems more to be a consistency check because  $\omega_Z$  was chosen to fulfill one of the zero force conditions automatically.

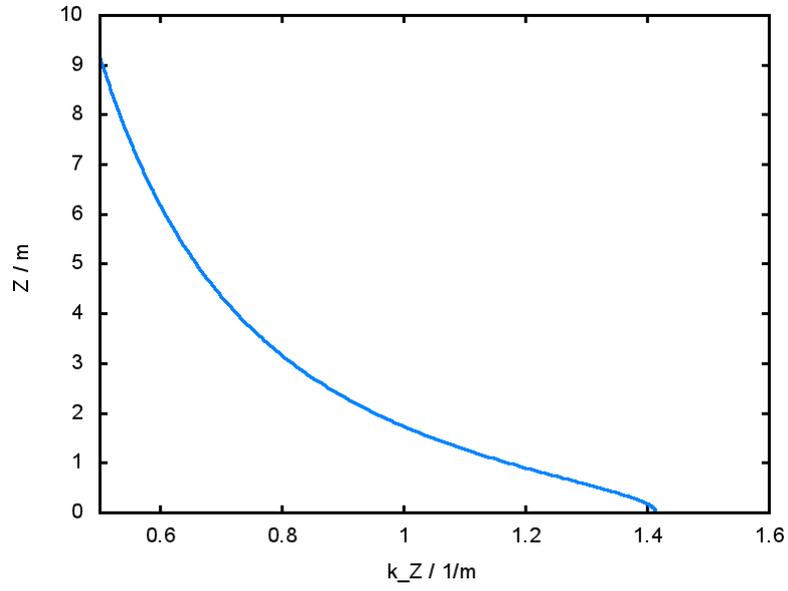


Figure 1:  $k_Z$  dependency of  $Z$  coordinate for linear  $\omega_Z$ .

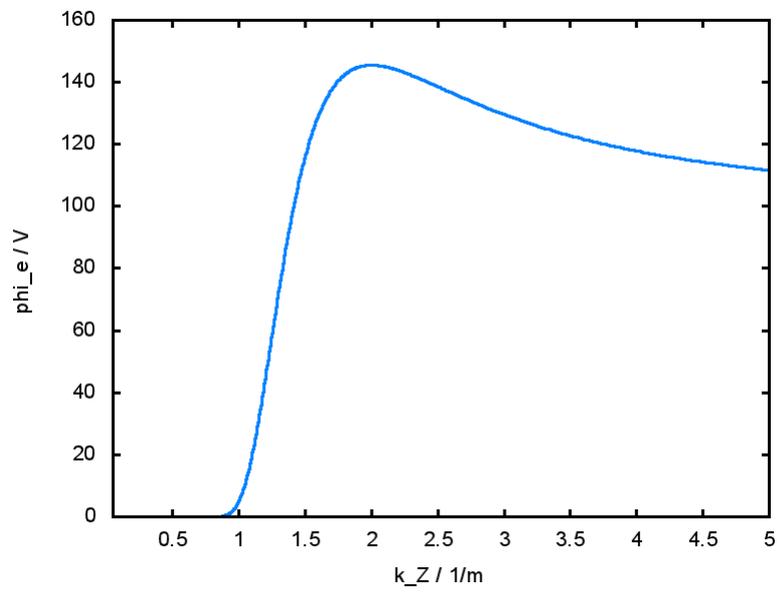


Figure 2:  $k_Z$  dependency of  $\phi_e$  for linear  $\omega_Z$ .

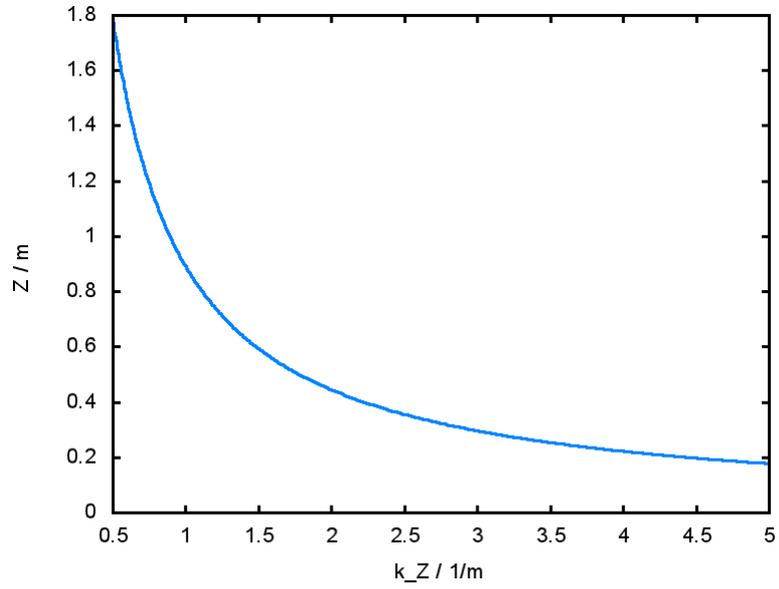


Figure 3:  $k_Z$  dependency of  $Z$  coordinate for hyperbolic  $\omega_Z$ .

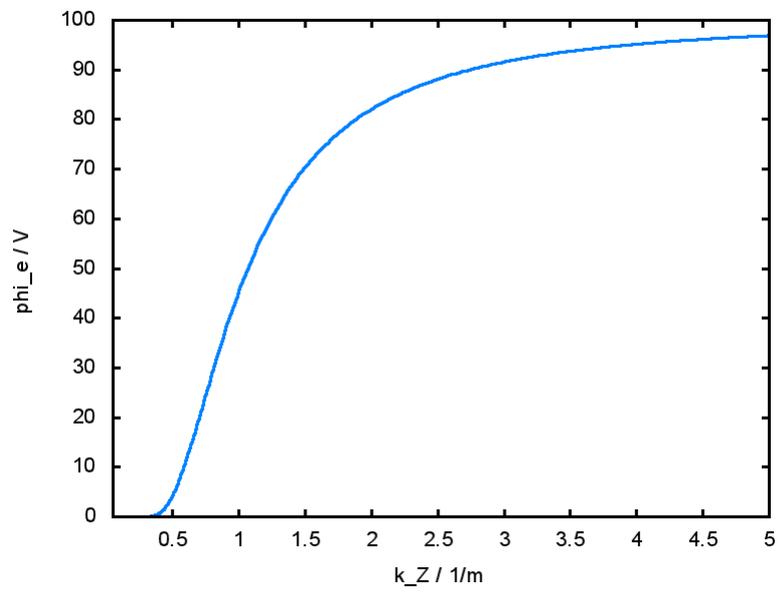


Figure 4:  $k_Z$  dependency of  $\phi_e$  for hyperbolic  $\omega_Z$ .

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{11} M .W. Evans and J. - P. Vigiér, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 2002, and Omnia Opera Section of [www.aias.us](http://www.aias.us)) in five volumes hardback, five volumes softback.

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