

(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
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(%o1) $f(i, j) := \text{sum}(R_{i, j, \sigma, 0} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 0}, \sigma, 0, 3) +$

$\text{sum}(R_{i, j, \sigma, 1} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 1}, \sigma, 0, 3) +$

$\text{sum}(R_{i, j, \sigma, 2} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 2}, \sigma, 0, 3) +$

$\text{sum}(R_{i, j, \sigma, 3} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 3}, \sigma, 0, 3)$

(%i2) /* define coordinate vector */

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array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
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(%o2) x

(%o3) [t , r , θ , ϕ]

(%i4) /* g1 is symm. metric with indices 1...4 */

```
g1: matrix(
  [-(1-2*M/r),0,0,-4*J/r*sin(theta)^2],
  [0,(1+2*M/r),0,0],
  [0,0,r^2,0],
  [-4*J/r*sin(theta)^2,0,0,r^2*sin(theta)^2]
);
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(%o4)
$$\begin{bmatrix} \frac{2M}{r} - 1 & 0 & 0 & -\frac{4 \sin(\theta)^2 J}{r} \\ 0 & \frac{2M}{r} + 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ -\frac{4 \sin(\theta)^2 J}{r} & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}$$

(%i5) /* contravariant g is inverse of g */

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gContr1: ratsimp(invert(g1));
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$$\begin{aligned}
 & \left(\% 5 \right) \left[\begin{array}{cccc} \frac{r^4}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} & 0 & 0 & \frac{4 r J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} \\ 0 & \frac{r}{2 M + r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ \frac{4 r J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} & 0 & 0 & \frac{2 r M - r^2}{2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} \end{array} \right]
 \end{aligned}$$

(%i6)

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/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g[mu,nu]: g1[mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$

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(%i7) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
            diff(g[nu,rho],x[mu]) +
            diff(g[rho,mu],x[nu]) -
            diff(g[mu,nu],x[rho])),
        rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$

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(%i8) /* display Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}}$

```

$$\Gamma_{0,0,1} = \frac{8 \sin(\theta)^2 J^2}{r(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{r^2 M}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,0,2} = - \frac{16 \cos(\theta) \sin(\theta) J^2}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,1,0} = \frac{8 \sin(\theta)^2 J^2}{r(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{r^2 M}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,1,3} = \frac{6 r^2 \sin(\theta)^2 J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,2,0} = - \frac{16 \cos(\theta) \sin(\theta) J^2}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,3,1} = \frac{6 r^2 \sin(\theta)^2 J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{1,0,0} = \frac{M}{r(2 M + r)}$$

$$\Gamma_{1,0,3} = - \frac{2 \sin(\theta)^2 J}{r(2 M + r)}$$

$$\Gamma_{1,1,1} = - \frac{M}{r(2 M + r)}$$

$$\Gamma_{1,2,2} = - \frac{r^2}{2 M + r}$$

$$\Gamma_{1,3,0} = - \frac{2 \sin(\theta)^2 J}{r(2 M + r)}$$

$$\Gamma_{1,3,3} = - \frac{r^2 \sin(\theta)^2}{2 M + r}$$

$$\Gamma_{2,0,3} = \frac{4 \cos(\theta) \sin(\theta) J}{r^3}$$

$$\Gamma_{2,1,2} = \frac{1}{r}$$

$$\Gamma_{2,2,1} = \frac{1}{r}$$

$$\Gamma_{2,3,0} = \frac{4 \cos(\theta) \sin(\theta) J}{r^3}$$

$$\Gamma_{2,3,3} = - \cos(\theta) \sin(\theta)$$

$$\Gamma_{3,0,1} = \frac{2 \sin(\theta)^2 J (2 r M - r^2)}{r^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - \frac{4 J M}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,0,2} = - \frac{4 \cos(\theta) \sin(\theta) J (2 r M - r^2)}{r (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}$$

$$\Gamma_{3,1,0} = \frac{2 \sin(\theta)^2 J (2 r M - r^2)}{r^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - \frac{4 J M}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,1,3} = \frac{r \sin(\theta)^2 (2 r M - r^2)}{2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} + \frac{8 \sin(\theta)^2 J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,2,0} = - \frac{4 \cos(\theta) \sin(\theta) J (2 r M - r^2)}{r (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}$$

$$\Gamma_{3,2,3} = \frac{r^2 \cos(\theta) \sin(\theta) (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{3,3,1} = \frac{r \sin(\theta)^2 (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} + \frac{8 \sin(\theta)^2 J^2}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,3,2} = \frac{r^2 \cos(\theta) \sin(\theta) (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

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(%i9) /* compute Riemann tensor elements */
for rho:0 thru 3 do {
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
  R[rho,sigma,mu,nu] :
  diff(Gamma[rho,nu,sigma],x[mu]) -
  diff(Gamma[rho,mu,sigma],x[nu]) +
  /* lambda sums by function call: */
  sum(
    Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
    Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
    lambda, 0, 3)
}}}}$
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```
(%i10) /* display R's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
for l:0 thru 3 do {
  R[i,j,k,l] : /ratsimp*/(factor(R[i,j,k,l])),
  if R[i,j,k,l] # 0 then display(R[i,j,k,l])
}}}}$
```

$$R_{0,0,0,3} = - \frac{4 \sin(\theta)^2 J (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r^3 (2M + r)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{0,0,1,2} = \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2M - r)}{(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,0,2,1} = - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2M - r)}{(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,0,3,0} = \frac{4 \sin(\theta)^2 J (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r^3 (2M + r)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{0,1,0,1} = -$$

$$\frac{2(2r^6 M^3 - 32r^3 \sin(\theta)^2 J^2 M^2 + 128 \sin(\theta)^4 J^4 M - r^8 M + 96 r \sin(\theta)^4 J^4 + 14 r^5 \sin(\theta)^2 J^2)}{r^2 (2M + r)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,0,2} = - \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,1,0} =$$

$$\frac{2 (2 r^6 M^3 - 32 r^3 \sin(\theta)^2 J^2 M^2 + 128 \sin(\theta)^4 J^4 M - r^8 M + 96 r \sin(\theta)^4 J^4 + 14 r^5 \sin(\theta)^2 J^2)}{r^2 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,1,3} = - \frac{6 r \sin(\theta)^2 J (48 \sin(\theta)^2 J^2 M + 16 r \sin(\theta)^2 J^2 - r^5)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,2,0} = \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,2,3} = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,3,1} = \frac{6 r \sin(\theta)^2 J (48 \sin(\theta)^2 J^2 M + 16 r \sin(\theta)^2 J^2 - r^5)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,3,2} = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,2,0,1} = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,2,0,2} = - (64 r^3 \sin(\theta)^2 J^2 M^2 - 64 r^3 \cos(\theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2) / ((2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{0,2,1,0} = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,2,1,3} = \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{0,2,2,0} = (64 r^3 \sin(\theta)^2 J^2 M^2 - 64 r^3 \cos(\theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2) / ((2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{0,2,2,3} = \frac{6 r^4 \sin(\theta)^2 J}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{0,2,3,1} = - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\begin{aligned}
R_{0,2,3,2} &= -\frac{6r^4 \sin(\theta)^2 J}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{0,3,0,3} &= \frac{\sin(\theta)^2(32\cos(\theta)^2J^2M+r^4M+4r\sin(\theta)^2J^2+16r\cos(\theta)^2J^2)}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{0,3,1,2} &= -\frac{6r^5\cos(\theta)\sin(\theta)J(2M-r)}{(2r^3M-16\sin(\theta)^2J^2-r^4)^2} \\
R_{0,3,2,1} &= \frac{6r^5\cos(\theta)\sin(\theta)J(2M-r)}{(2r^3M-16\sin(\theta)^2J^2-r^4)^2} \\
R_{0,3,3,0} &= -\frac{\sin(\theta)^2(32\cos(\theta)^2J^2M+r^4M+4r\sin(\theta)^2J^2+16r\cos(\theta)^2J^2)}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,0,1} &= \frac{2(2r^3M^3-16\sin(\theta)^2J^2M^2-8r\sin(\theta)^2J^2M-r^5M+2r^2\sin(\theta)^2J^2)}{r^2(2M+r)^2(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,0,2} &= -\frac{8\cos(\theta)\sin(\theta)J^2}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,1,0} &= -\frac{2(2r^3M^3-16\sin(\theta)^2J^2M^2-8r\sin(\theta)^2J^2M-r^5M+2r^2\sin(\theta)^2J^2)}{r^2(2M+r)^2(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,1,3} &= \frac{2\sin(\theta)^2J(4r^3M^2-32\sin(\theta)^2J^2M-2r^4M-24r\sin(\theta)^2J^2-3r^5)}{r^2(2M+r)^2(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,2,0} &= \frac{8\cos(\theta)\sin(\theta)J^2}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,2,3} &= -\frac{12\cos(\theta)\sin(\theta)J(2r^3M-8\sin(\theta)^2J^2-r^4)}{r(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,3,1} &= -\frac{2\sin(\theta)^2J(4r^3M^2-32\sin(\theta)^2J^2M-2r^4M-24r\sin(\theta)^2J^2-3r^5)}{r^2(2M+r)^2(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,0,3,2} &= \frac{12\cos(\theta)\sin(\theta)J(2r^3M-8\sin(\theta)^2J^2-r^4)}{r(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,2,0,3} &= -\frac{6r^2\cos(\theta)\sin(\theta)J(2M-r)}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)} \\
R_{1,2,1,2} &= -\frac{rM}{(2M+r)^2} \\
R_{1,2,2,1} &= \frac{rM}{(2M+r)^2} \\
R_{1,2,3,0} &= \frac{6r^2\cos(\theta)\sin(\theta)J(2M-r)}{(2M+r)(2r^3M-16\sin(\theta)^2J^2-r^4)}
\end{aligned}$$

$$R_{1,3,0,1} = - \frac{2 \sin(\theta)^2 J (4 r^3 M^2 - 32 \sin(\theta)^2 J^2 M - 2 r^4 M - 24 r \sin(\theta)^2 J^2 - 3 r^5)}{r^2 (2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,3,0,2} = \frac{6 \cos(\theta) \sin(\theta) J}{r (2 M + r)}$$

$$R_{1,3,1,0} = \frac{2 \sin(\theta)^2 J (4 r^3 M^2 - 32 \sin(\theta)^2 J^2 M - 2 r^4 M - 24 r \sin(\theta)^2 J^2 - 3 r^5)}{r^2 (2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,3,1,3} = - \frac{r \sin(\theta)^2 (2 r^3 M^2 - 88 \sin(\theta)^2 J^2 M - r^4 M - 36 r \sin(\theta)^2 J^2)}{(2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,3,2,0} = - \frac{6 \cos(\theta) \sin(\theta) J}{r (2 M + r)}$$

$$R_{1,3,3,1} = \frac{r \sin(\theta)^2 (2 r^3 M^2 - 88 \sin(\theta)^2 J^2 M - r^4 M - 36 r \sin(\theta)^2 J^2)}{(2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,0,1} = - \frac{8 \cos(\theta) \sin(\theta) J^2}{r^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,0,2} = - \frac{64 \cos(\theta)^2 J^2 M^2 + 2 r^4 M^2 - 16 r \sin(\theta)^2 J^2 M - r^5 M - 16 r^2 \cos(\theta)^2 J^2}{r^3 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,1,0} = \frac{8 \cos(\theta) \sin(\theta) J^2}{r^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,1,3} = - \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

$$R_{2,0,2,0} = \frac{64 \cos(\theta)^2 J^2 M^2 + 2 r^4 M^2 - 16 r \sin(\theta)^2 J^2 M - r^5 M - 16 r^2 \cos(\theta)^2 J^2}{r^3 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,2,3} = - \frac{2 \sin(\theta)^2 J (4 M + 3 r)}{r^3 (2 M + r)}$$

$$R_{2,0,3,1} = \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

$$R_{2,0,3,2} = \frac{2 \sin(\theta)^2 J (4 M + 3 r)}{r^3 (2 M + r)}$$

$$R_{2,1,0,3} = \frac{6 \cos(\theta) \sin(\theta) J (2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,1,1,2} = \frac{M}{r^2 (2 M + r)}$$

$$R_{2,1,2,1} = - \frac{M}{r^2 (2 M + r)}$$

$$R_{2,1,3,0} = - \frac{6 \cos(\theta) \sin(\theta) J (2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,3,0,1} = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,3,0,2} = \frac{2 \sin(\theta)^2 J(4 M + 3 r)}{r^3 (2 M + r)}$$

$$R_{2,3,1,0} = - \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,3,2,0} = - \frac{2 \sin(\theta)^2 J(4 M + 3 r)}{r^3 (2 M + r)}$$

$$R_{2,3,2,3} = \frac{2 \sin(\theta)^2 M}{2 M + r}$$

$$R_{2,3,3,2} = - \frac{2 \sin(\theta)^2 M}{2 M + r}$$

$$R_{3,0,0,3} = - \frac{(2 M - r)(32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r^3 (2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,0,1,2} = \frac{6 r^2 \cos(\theta) J(2 M - r)^2}{\sin(\theta)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,0,2,1} = - \frac{6 r^2 \cos(\theta) J(2 M - r)^2}{\sin(\theta)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,0,3,0} = \frac{(2 M - r)(32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r^3 (2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,1,0,1} = - \frac{2 J(8 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - 16 r \sin(\theta)^2 J^2 - 3 r^5)}{r(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,0,2} = - \frac{2 \cos(\theta) J(12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,1,0} = \frac{2 J(8 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - 16 r \sin(\theta)^2 J^2 - 3 r^5)}{r(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,1,3} = (4 r^6 M^3 - 208 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M + 192 r \sin(\theta)^4 J^4 + 60 r^5 \sin(\theta)^2 J^2) / (r^2 (2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{3,1,2,0} = \frac{2 \cos(\theta) J(12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,2,3} = \frac{48 \cos(\theta) \sin(\theta) J^2(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,3,1} = - (4 r^6 M^3 - 208 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M + 192 r \sin(\theta)^4 J^4 + 60 r^5 \sin(\theta)^2 J^2) / (r^2 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{3,1,3,2} = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,0,1} = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,0,2} = - (2 J (16 r^3 M^3 - 128 \sin(\theta)^2 J^2 M^2 - 128 \cos(\theta)^2 J^2 M^2 - 8 r^4 M^2 - 6 r^5 M + 48 r^2 \sin(\theta)^2 J^2 + 32 r^2 \cos(\theta)^2 J^2 + 3 r^6)) / ((2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{3,2,1,0} = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,1,3} = \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,2,2,0} = (2 J (16 r^3 M^3 - 128 \sin(\theta)^2 J^2 M^2 - 128 \cos(\theta)^2 J^2 M^2 - 8 r^4 M^2 - 6 r^5 M + 48 r^2 \sin(\theta)^2 J^2 + 32 r^2 \cos(\theta)^2 J^2 + 3 r^6)) / ((2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{3,2,2,3} = - \frac{2 (2 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,2,3,1} = - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,2,3,2} = \frac{2 (2 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,3,0,3} = \frac{4 \sin(\theta)^2 J (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r^3 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,3,1,2} = - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,3,2,1} = \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,3,3,0} = - \frac{4 \sin(\theta)^2 J (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r^3 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

```
(%i11) /* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

```
(%i12) /* display Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
if Ric[i,j] # 0 then display(Ric[i,j])
}}$
```

$$Ric_{0,0} = (4 (64 \cos(\theta)^2 J^2 M^3 + r^4 M^3 + 4 r \sin(\theta)^2 J^2 M^2 + 32 r \cos(\theta)^2 J^2 M^2 - 16 r^2 \cos(\theta)^2 J^2 M - 2 r^3 \sin(\theta)^2 J^2 - 8 r^3 \cos(\theta)^2 J^2)) / (r^3 (2M+r)^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4))$$

$$Ric_{0,3} = - (8 \sin(\theta)^2 J (4 r^3 M^3 - 32 \sin(\theta)^2 J^2 M^2 + 32 \cos(\theta)^2 J^2 M^2 + 3 r^4 M^2 - 28 r \sin(\theta)^2 J^2 M + 32 r \cos(\theta)^2 J^2 M - 4 r^2 \sin(\theta)^2 J^2 + 8 r^2 \cos(\theta)^2 J^2)) / (r^3 (2M+r)^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4))$$

$$Ric_{1,1} = - (4 (3 r^6 M^3 - 84 r^3 \sin(\theta)^2 J^2 M^2 - 2 r^7 M^2 + 192 \sin(\theta)^4 J^4 M + 16 r^4 \sin(\theta)^2 J^2 M + 96 r \sin(\theta)^4 J^4 + 22 r^5 \sin(\theta)^2 J^2)) / (r^2 (2M+r) (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$Ric_{1,2} = - \frac{16 \cos(\theta) \sin(\theta) J^2 (9 r^3 M - 48 \sin(\theta)^2 J^2 - 5 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$Ric_{2,1} = - \frac{16 \cos(\theta) \sin(\theta) J^2 (9 r^3 M - 48 \sin(\theta)^2 J^2 - 5 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$Ric_{2,2} = (8 (2 r^6 M^4 - 48 r^3 \sin(\theta)^2 J^2 M^3 + 16 r^3 \cos(\theta)^2 J^2 M^3 - r^7 M^3 + 256 \sin(\theta)^4 J^4 M^2 - 20 r^4 \sin(\theta)^2 J^2 M^2 + 8 r^4 \cos(\theta)^2 J^2 M^2 + 288 r \sin(\theta)^4 J^4 M + 6 r^5 \sin(\theta)^2 J^2 M - 4 r^5 \cos(\theta)^2 J^2 M + 96 r^2 \sin(\theta)^4 J^4 + 6 r^6 \sin(\theta)^2 J^2 - 2 r^6 \cos(\theta)^2 J^2)) / ((2M+r)^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$Ric_{3,0} = - (8 \sin(\theta)^2 J (4 r^3 M^3 - 32 \sin(\theta)^2 J^2 M^2 + 32 \cos(\theta)^2 J^2 M^2 + 3 r^4 M^2 - 28 r \sin(\theta)^2 J^2 M + 32 r \cos(\theta)^2 J^2 M - 4 r^2 \sin(\theta)^2 J^2 + 8 r^2 \cos(\theta)^2 J^2)) / (r^3 (2M+r)^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4))$$

$$Ric_{3,3} = (8 \sin(\theta)^2 (r^3 M^3 - 8 \sin(\theta)^2 J^2 M^2 + 8 \cos(\theta)^2 J^2 M^2 + 8 r \sin(\theta)^2 J^2 M + 8 r \cos(\theta)^2 J^2 M + 5 r^2 \sin(\theta)^2 J^2 + 2 r^2 \cos(\theta)^2 J^2)) / ((2M+r)^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4))$$

```
(%i13) /* Ricci Scalar */
```

```
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
      + sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
      + sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
      + sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
      ;
```

```
(%o13) ( 8 ( 2 r^6 M^4 - 48 r^3 sin(theta)^2 J^2 M^3 + 16 r^3 cos(theta)^2 J^2 M^3 - r^7 M^3 +
256 sin(theta)^4 J^4 M^2 - 20 r^4 sin(theta)^2 J^2 M^2 + 8 r^4 cos(theta)^2 J^2 M^2 + 288 r sin(theta)^4
J^4 M + 6 r^5 sin(theta)^2 J^2 M - 4 r^5 cos(theta)^2 J^2 M + 96 r^2 sin(theta)^4 J^4 + 6 r^6
sin(theta)^2 J^2 - 2 r^6 cos(theta)^2 J^2 ) ) / ( r^2 (2 M + r)^2 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2 )
+ ( 4 r ( 64 cos(theta)^2 J^2 M^3 + r^4 M^3 + 4 r sin(theta)^2 J^2 M^2 + 32 r cos(theta)^2 J^2 M^2 -
16 r^2 cos(theta)^2 J^2 M - 2 r^3 sin(theta)^2 J^2 - 8 r^3 cos(theta)^2 J^2 ) ) / ( (2 M + r)^2
(2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2 ) - ( 4 ( 3 r^6 M^3 - 84 r^3 sin(theta)^2 J^2 M^2 - 2 r^7 M^2 +
192 sin(theta)^4 J^4 M + 16 r^4 sin(theta)^2 J^2 M + 96 r sin(theta)^4 J^4 + 22 r^5 sin(theta)^2 J^2 )
) / ( r (2 M + r)^2 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2 ) - ( 64 sin(theta)^2 J^2 ( 4 r^3 M^3 - 32
sin(theta)^2 J^2 M^2 + 32 cos(theta)^2 J^2 M^2 + 3 r^4 M^2 - 28 r sin(theta)^2 J^2 M + 32 r cos(theta)^2
J^2 M - 4 r^2 sin(theta)^2 J^2 + 8 r^2 cos(theta)^2 J^2 ) ) / ( r^2 (2 M + r)^2
(2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2 ) + ( 8 sin(theta)^2 (2 r M - r^2) ( r^3 M^3 - 8 sin(theta)^2 J^2
M^2 + 8 cos(theta)^2 J^2 M^2 + 8 r sin(theta)^2 J^2 M + 8 r cos(theta)^2 J^2 M + 5 r^2 sin(theta)^2 J^2 +
2 r^2 cos(theta)^2 J^2 ) ) / ( (2 M + r)^2 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)
(2 r^3 sin(theta)^2 M - 16 sin(theta)^4 J^2 - r^4 sin(theta)^2 ) )
```

```
(%i14) ratsimp(RicSc);
```

```
(%o14) ( 32 r^6 M^4 + ((512 r^3 cos(theta)^2 - 768 r^3 sin(theta)^2) J^2 - 24 r^7) M^3 +
((4096 sin(theta)^4 - 2048 cos(theta)^2 sin(theta)^2) J^4 + (192 r^4 sin(theta)^2 + 256 r^4 cos(theta)^2) J^2 + 8 r^8)
M^2 + ((3328 r sin(theta)^4 - 2048 r cos(theta)^2 sin(theta)^2) J^4 - 128 r^5 cos(theta)^2 J^2) M +
(640 r^2 sin(theta)^4 - 512 r^2 cos(theta)^2 sin(theta)^2) J^4 + (- 88 r^6 sin(theta)^2 - 64 r^6 cos(theta)^2) J^2
) / ( 16 r^8 M^4 - 256 r^5 sin(theta)^2 J^2 M^3 +
(1024 r^2 sin(theta)^4 J^4 - 128 r^6 sin(theta)^2 J^2 - 8 r^10) M^2 +
(1024 r^3 sin(theta)^4 J^4 + 64 r^7 sin(theta)^2 J^2) M + 256 r^4 sin(theta)^4 J^4 + 32 r^8 sin(theta)^2 J^2
+ r^12 )
```

```
(%i47)
```

```

/* Test for R^q */
for mu: 0 thru 3 do (
for sigma:0 thru 3 do (
for nu: 0 thru 3 do (
for rho: 0 thru 3 do (
  R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
  if R_q # 0 then (
    display("====Einstein equation R^q=0 not fulfilled! "),
    display(mu,sigma,nu,rho),
    display(R_q)
  )
)))));

```

====Einstein equation R^q=0 not fulfilled! =

====Einstein equation R^q=0 not fulfilled!

$$\mu = 0$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = -\frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

====Einstein equation R^q=0 not fulfilled! =

====Einstein equation R^q=0 not fulfilled!

$$\mu = 0$$

$$\sigma = 0$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

====Einstein equation R^q=0 not fulfilled! =

====Einstein equation R^q=0 not fulfilled!

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 2$$

$$R_q = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 0$$

$$R_q = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 3$$

$$R_q = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} -$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 3$$

$$\rho = 2$$

$$R_q = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} +$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 0$$

$$\rho = 1$$

$$R_q = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 0$$

$$R_q = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 3$$

$$R_q = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} + \frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 3$$

$$\rho = 1$$

$$R_q = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} -$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 3$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} -$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 3$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} +$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 2$$

$$\rho = 3$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 r^2 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} +$$

$$+ \frac{6 \cos(\theta) \sin(\theta) J}{r (2 M + r)}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 3$$

$$\rho = 2$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 r^2 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r(2 M + r)}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 2$$

$$\nu = 0$$

$$\rho = 3$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 r^2 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r(2 M + r)}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 2$$

$$\nu = 3$$

$$\rho = 0$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 r^2 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J}{r(2 M + r)}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 3$$

$$\nu = 0$$

$$\rho = 2$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 r^2 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J}{r (2 M + r)}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 3$$

$$\nu = 2$$

$$\rho = 0$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 r^2 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r (2 M + r)}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 3$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J (2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 0$$

$$\nu = 3$$

$$\rho = 1$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J (2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 1$$

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$$\rho = 3$$

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=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 1$$

$$\nu = 3$$

$$\rho = 0$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 3$$

$$\nu = 0$$

$$\rho = 1$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

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$$\mu = 2$$

$$\sigma = 3$$

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$$\rho = 0$$

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=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 0$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

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$$\mu = 3$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 2$$

$$R_q = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

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$\mu = 3$

$\sigma = 1$

$\nu = 2$

$\rho = 0$

$$R_q = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$\frac{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$\mu = 3$

$\sigma = 1$

$\nu = 2$

$\rho = 3$

$$R_q = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$\mu = 3$

$\sigma = 1$

$\nu = 3$

$\rho = 2$

$$R_q = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} +$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 0$$

$$\rho = 1$$

$$R_q = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 0$$

$$R_q = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 3$$

$$R_q = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} +$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 3$$

$$\rho = 1$$

$$R_q = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 3$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 3$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} +$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

(%o47) done

```
(%i15) /* Raising of indices,
        contravariant metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/
```

```
R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);
```

$$(\%o15) \quad \frac{24 r^3 \sin(\theta)^2 J^2 (48 \sin(\theta)^2 J^2 M + 16 r \sin(\theta)^2 J^2 - r^5)}{(2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3} -$$

$$\frac{2 r^3 (2 r^6 M^3 - 32 r^3 \sin(\theta)^2 J^2 M^2 + 128 \sin(\theta)^4 J^4 M - r^8 M + 96 r \sin(\theta)^4 J^4 + 14 r^5 \sin(\theta)^2 J^2)}{(2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3}$$

$$(\%o16) \quad - (r^2 (64 r^3 \sin(\theta)^2 J^2 M^2 - 64 r^3 \cos(\theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2)) / ((2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3) -$$

$$\frac{24 r^3 \sin(\theta)^2 J^2}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$(\%o17) \quad \frac{r^4 \sin(\theta)^2 (2 r M - r^2) (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} -$$

$$\frac{16 r^2 \sin(\theta)^2 J^2 (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3}$$

```
(%i18) R0101: factor(R0101);
        R0202: factor(R0202);
        R0303: factor(R0303);
```

```
(%o18) -
```

$$2 r^3 (2 r^6 M^3 - 32 r^3 \sin(\theta)^2 J^2 M^2 - 448 \sin(\theta)^4 J^4 M - r^8 M - 96 r \sin(\theta)^4 J^4 + 26 r^5 \sin(\theta)^2 J^2)$$

$$(2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3$$

$$(\%o19) \quad - (r^2 (64 r^3 \sin(\theta)^2 J^2 M^2 - 64 r^3 \cos(\theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 \sin(\theta)^4 J^4 M + 80 r^4 \sin(\theta)^2 J^2 M + r^8 M - 768 r \sin(\theta)^4 J^4 - 48 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2)) / ((2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3)$$

$$(\%o20) \quad \frac{r^2 (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

```
(%i21) R1010: f(1,0);
        R1212: f(1,2);
        R1313: f(1,3);
```

$$\begin{aligned}
 (\%o21) \quad & \frac{8 \sin(\theta)^2 J^2 (4 r^3 M^2 - 32 \sin(\theta)^2 J^2 M - 2 r^4 M - 24 r \sin(\theta)^2 J^2 - 3 r^5)}{(2 M + r)^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \\
 & \frac{2 r^3 (2 r^3 M^3 - 16 \sin(\theta)^2 J^2 M^2 - 8 r \sin(\theta)^2 J^2 M - r^5 M + 2 r^2 \sin(\theta)^2 J^2)}{(2 M + r)^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}
 \end{aligned}$$

$$(\%o22) \quad - \frac{M}{(2 M + r)^3}$$

$$\begin{aligned}
 (\%o23) \quad & \frac{8 \sin(\theta)^2 J^2 (4 r^3 M^2 - 32 \sin(\theta)^2 J^2 M - 2 r^4 M - 24 r \sin(\theta)^2 J^2 - 3 r^5)}{(2 M + r)^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \\
 & \frac{r^2 \sin(\theta)^2 (2 r M - r^2) (2 r^3 M^2 - 88 \sin(\theta)^2 J^2 M - r^4 M - 36 r \sin(\theta)^2 J^2)}{(2 M + r)^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4) (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}
 \end{aligned}$$

(%i24) `R1010: factor(R1010);`
`R1212: factor(R1212);`
`R1313: factor(R1313);`

$$\begin{aligned}
 (\%o24) \quad & - \\
 & \frac{2 (2 r^6 M^3 - 32 r^3 \sin(\theta)^2 J^2 M^2 + 128 \sin(\theta)^4 J^4 M - r^8 M + 96 r \sin(\theta)^4 J^4 + 14 r^5 \sin(\theta)^2 J^2)}{(2 M + r)^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}
 \end{aligned}$$

$$(\%o25) \quad - \frac{M}{(2 M + r)^3}$$

$$\begin{aligned}
 (\%o26) \quad & - (4 r^6 M^3 - 208 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 32 \\
 & r^4 \sin(\theta)^2 J^2 M + r^8 M + 192 r \sin(\theta)^4 J^4 + 60 r^5 \sin(\theta)^2 J^2) / ((2 M + r)^3 \\
 & (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)
 \end{aligned}$$

(%i27) `R2020: f(2,0);`
`R2121: f(2,1);`
`R2323: f(2,3);`

$$\begin{aligned}
 (\%o27) \quad & \frac{64 \cos(\theta)^2 J^2 M^2 + 2 r^4 M^2 - 16 r \sin(\theta)^2 J^2 M - r^5 M - 16 r^2 \cos(\theta)^2 J^2}{r (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \\
 & \frac{8 \sin(\theta)^2 J^2 (4 M + 3 r)}{r^4 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}
 \end{aligned}$$

$$(\%o28) \quad - \frac{M}{r^3 (2 M + r)^2}$$

$$\begin{aligned}
 (\%o29) \quad & \frac{2 \sin(\theta)^2 M (2 r M - r^2)}{r^2 (2 M + r) (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - \\
 & \frac{8 \sin(\theta)^2 J^2 (4 M + 3 r)}{r^4 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}
 \end{aligned}$$


```
(%i30) R2020: factor(R2020);
R2121: factor(R2121);
R2323: factor(R2323);
```

$$(\%o30) \quad - (64 r^3 \sin(\theta)^2 J^2 M^2 - 64 r^3 \cos(\theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2) / (r^4 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$(\%o31) \quad - \frac{M}{r^3 (2 M + r)^2}$$

$$(\%o32) \quad \frac{2 (2 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{r^4 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

```
(%i33) R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);
```

$$(\%o33) \quad \frac{r (2 M - r) (2 r M - r^2) (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{(2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - \frac{16 J^2 (2 M - r) (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3}$$

$$(\%o34) \quad - ((2 r M - r^2) (4 r^6 M^3 - 208 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 32 r^4 \sin(\theta)^2 J^2 M + r^8 M + 192 r \sin(\theta)^4 J^4 + 60 r^5 \sin(\theta)^2 J^2)) / (r (2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)) - \frac{8 r J^2 (8 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - 16 r \sin(\theta)^2 J^2 - 3 r^5)}{(2 M + r)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3}$$

$$(\%o35) \quad \frac{2 (2 r M - r^2) (2 r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{r^2 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4) (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - (8 J^2 (16 r^3 M^3 - 128 \sin(\theta)^2 J^2 M^2 - 128 \cos(\theta)^2 J^2 M^2 - 8 r^4 M^2 - 6 r^5 M + 48 r^2 \sin(\theta)^2 J^2 + 32 r^2 \cos(\theta)^2 J^2 + 3 r^6)) / (r (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3)$$

```
(%i36) R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);
```

$$(\%o36) \quad \frac{(2 M - r) (32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2)}{r \sin(\theta)^2 (2 M + r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

```
(%o37) - ( 8 r^6 M^4 - 416 r^3 sin(theta)^2 J^2 M^3 - 12 r^7 M^3 + 512 sin(theta)^4 J^4 M^2 +
336 r^4 sin(theta)^2 J^2 M^2 + 6 r^8 M^2 + 88 r^5 sin(theta)^2 J^2 M - r^9 M - 320 r^2 sin(theta)^4
J^4 - 84 r^6 sin(theta)^2 J^2 ) / ( sin(theta)^2 (2 M + r)^2 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^3 )
```

```
(%o38) ( 2 ( 8 r^6 M^4 - 192 r^3 sin(theta)^2 J^2 M^3 - 12 r^7 M^3 + 1024 sin(theta)^4 J^4 M^2
+ 512 cos(theta)^2 sin(theta)^2 J^4 M^2 + 112 r^4 sin(theta)^2 J^2 M^2 + 6 r^8 M^2 + 128 r sin(theta)^4
J^4 M + 40 r^5 sin(theta)^2 J^2 M - r^9 M - 384 r^2 sin(theta)^4 J^4 - 128 r^2 cos(theta)^2 sin(theta)^2
J^4 - 24 r^6 sin(theta)^2 J^2 ) ) / ( r sin(theta)^2 (2 M + r) (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^3 )
```

```
(%i39) /* Coulomb law */
DivE : R0101 + R0202 + R0303;
```

```
(%o39) -
2 r^3 (2 r^6 M^3 - 32 r^3 sin(theta)^2 J^2 M^2 - 448 sin(theta)^4 J^4 M - r^8 M - 96 r sin(theta)^4 J^4 + 26 r^5 sin(theta)^2 J^2)
-----
(2 M + r)^2 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^3
- ( r^2 ( 64 r^3 sin(theta)^2 J^2 M^2 - 64 r^3 cos(theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 sin(theta)^4 J^4
M + 80 r^4 sin(theta)^2 J^2 M + r^8 M - 768 r sin(theta)^4 J^4 - 48 r^5 sin(theta)^2 J^2 + 16 r^5
cos(theta)^2 J^2 ) ) / ( (2 M + r) (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^3 ) +
r^2 (32 cos(theta)^2 J^2 M + r^4 M + 4 r sin(theta)^2 J^2 + 16 r cos(theta)^2 J^2)
-----
(2 M + r) (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2
```

```
(%i40) ratsimp(DivE);
```

```
(%o40) - ( ((128 r^5 sin(theta)^2 - 256 r^5 cos(theta)^2) J^2 - 4 r^9) M^3 +
((1024 r^2 cos(theta)^2 sin(theta)^2 - 1024 r^2 sin(theta)^4) J^4 + (176 r^6 sin(theta)^2 - 128 r^6 cos(theta)^2) J^2)
M^2 + ((1024 r^3 cos(theta)^2 sin(theta)^2 - 2816 r^3 sin(theta)^4) J^4 + 64 r^7 cos(theta)^2 J^2) M +
(256 r^4 cos(theta)^2 sin(theta)^2 - 896 r^4 sin(theta)^4) J^4 + (8 r^8 sin(theta)^2 + 32 r^8 cos(theta)^2) J^2 ) /
( 32 r^9 M^5 + (- 768 r^6 sin(theta)^2 J^2 - 16 r^10) M^4 + (6144 r^3 sin(theta)^4 J^4 - 16 r^11) M^3 +
(- 16384 sin(theta)^6 J^6 + 3072 r^4 sin(theta)^4 J^4 + 384 r^8 sin(theta)^2 J^2 + 8 r^12) M^2 +
(- 16384 r sin(theta)^6 J^6 - 1536 r^5 sin(theta)^4 J^4 + 2 r^13) M - 4096 r^2 sin(theta)^6 J^6 - 768
r^6 sin(theta)^4 J^4 - 48 r^10 sin(theta)^2 J^2 - r^14 )
```

```
(%i41) /* J[r] */
Jr : -(R1010 + R1212 + R1313);
```

```
(%o41) ( 4 r^6 M^3 - 208 r^3 sin(theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 sin(theta)^4 J^4 M + 32 r^4
sin(theta)^2 J^2 M + r^8 M + 192 r sin(theta)^4 J^4 + 60 r^5 sin(theta)^2 J^2 ) / ( (2 M + r)^3
(2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2 ) +
```

$$\frac{2(2r^6M^3 - 32r^3\sin(\theta)^2J^2M^2 + 128\sin(\theta)^4J^4M - r^8M + 96r\sin(\theta)^4J^4 + 14r^5\sin(\theta)^2J^2)}{(2M+r)^3(2r^3M - 16\sin(\theta)^2J^2 - r^4)^2} + \frac{M}{(2M+r)^3}$$

```
(%i42) ratsimp(Jr);
```

```
(%o42) ( 12 r^6 M^3 + (- 336 r^3 sin(theta)^2 J^2 - 8 r^7) M^2 +
(768 sin(theta)^4 J^4 + 64 r^4 sin(theta)^2 J^2) M + 384 r sin(theta)^4 J^4 + 88 r^5 sin(theta)^2 J^2 ) / (
32 r^6 M^5 + (16 r^7 - 512 r^3 sin(theta)^2 J^2) M^4 +
(2048 sin(theta)^4 J^4 - 512 r^4 sin(theta)^2 J^2 - 16 r^8) M^3 + (3072 r sin(theta)^4 J^4 - 8 r^9) M^2 +
(1536 r^2 sin(theta)^4 J^4 + 128 r^6 sin(theta)^2 J^2 + 2 r^10) M + 256 r^3 sin(theta)^4 J^4 + 32 r^7
sin(theta)^2 J^2 + r^11 )
```

```
(%i43) /* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);
```

```
(%o43) ( 64 r^3 sin(theta)^2 J^2 M^2 - 64 r^3 cos(theta)^2 J^2 M^2 - 2 r^7 M^2 - 512 sin(theta)^4
J^4 M + 32 r^4 sin(theta)^2 J^2 M + r^8 M - 384 r sin(theta)^4 J^4 - 24 r^5 sin(theta)^2 J^2 + 16 r^5
cos(theta)^2 J^2 ) / ( r^4 (2M+r)(2r^3M - 16 sin(theta)^2 J^2 - r^4)^2 ) -
2(2r^3M^2 - 16 sin(theta)^2 J^2 M - r^4 M - 12 r sin(theta)^2 J^2) / (r^4(2M+r)(2r^3M - 16 sin(theta)^2 J^2 - r^4)) + M / (r^3(2M+r)^2)
```

```
(%i44) ratsimp(Jtheta);
```

```
(%o44) - ( 16 r^6 M^4 + ((128 r^3 cos(theta)^2 - 384 r^3 sin(theta)^2)J^2 - 8 r^7) M^3 +
(2048 sin(theta)^4 J^4 + (64 r^4 cos(theta)^2 - 160 r^4 sin(theta)^2)J^2) M^2 +
(2304 r sin(theta)^4 J^4 + (48 r^5 sin(theta)^2 - 32 r^5 cos(theta)^2)J^2) M + 768 r^2 sin(theta)^4 J^4 +
(48 r^6 sin(theta)^2 - 16 r^6 cos(theta)^2) J^2 ) / ( 16 r^10 M^4 - 256 r^7 sin(theta)^2 J^2 M^3 +
(1024 r^4 sin(theta)^4 J^4 - 128 r^8 sin(theta)^2 J^2 - 8 r^12) M^2 +
(1024 r^5 sin(theta)^4 J^4 + 64 r^9 sin(theta)^2 J^2) M + 256 r^6 sin(theta)^4 J^4 + 32 r^10 sin(theta)^2 J^2
+ r^14 )
```

```
(%i45) /* J[phi] */
Jphi : -(R3030 + R3131 + R3232);
```

```
(%o45) - ( 2 ( 8 r^6 M^4 - 192 r^3 sin(theta)^2 J^2 M^3 - 12 r^7 M^3 + 1024 sin(theta)^4 J^4
M^2 + 512 cos(theta)^2 sin(theta)^2 J^4 M^2 + 112 r^4 sin(theta)^2 J^2 M^2 + 6 r^8 M^2 + 128 r
sin(theta)^4 J^4 M + 40 r^5 sin(theta)^2 J^2 M - r^9 M - 384 r^2 sin(theta)^4 J^4 - 128 r^2 cos(theta)^2
```

$$\frac{\begin{aligned} & \sin(\theta)^2 J^4 - 24 r^6 \sin(\theta)^2 J^2 \big) \big) / \big(r \sin(\theta)^2 (2M + r) \\ & (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 \big) + \big(8 r^6 M^4 - 416 r^3 \sin(\theta)^2 J^2 M^3 - 12 r^7 M^3 + \\ & 512 \sin(\theta)^4 J^4 M^2 + 336 r^4 \sin(\theta)^2 J^2 M^2 + 6 r^8 M^2 + 88 r^5 \sin(\theta)^2 J^2 M - r^9 \\ & M - 320 r^2 \sin(\theta)^4 J^4 - 84 r^6 \sin(\theta)^2 J^2 \big) / \big(\sin(\theta)^2 (2M + r)^2 \\ & (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 \big) - \\ & (2M - r)(32 \cos(\theta)^2 J^2 M + r^4 M + 4 r \sin(\theta)^2 J^2 + 16 r \cos(\theta)^2 J^2) \end{aligned}}{r \sin(\theta)^2 (2M + r)(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

(%i46) `ev(ratsimp(Jphi),r);`

$$\begin{aligned} & (%o46) \quad - \big(32 r^6 M^5 + ((256 r^3 \cos(\theta)^2 - 768 r^3 \sin(\theta)^2) J^2 - 32 r^7) M^4 + \\ & (4096 \sin(\theta)^4 J^4 + 448 r^4 \sin(\theta)^2 J^2 + 8 r^8) M^3 + \\ & (1792 r \sin(\theta)^4 J^4 + (32 r^5 \sin(\theta)^2 - 128 r^5 \cos(\theta)^2) J^2) M^2 + \\ & (-1280 r^2 \sin(\theta)^4 J^4 - 96 r^6 \sin(\theta)^2 J^2) M - 384 r^3 \sin(\theta)^4 J^4 + \\ & (40 r^7 \sin(\theta)^2 + 16 r^7 \cos(\theta)^2) J^2 \big) / \big(32 r^{10} \sin(\theta)^2 M^5 + \\ & (-768 r^7 \sin(\theta)^4 J^2 - 16 r^{11} \sin(\theta)^2) M^4 + (6144 r^4 \sin(\theta)^6 J^4 - 16 r^{12} \sin(\theta)^2) M^3 \\ & + (-16384 r \sin(\theta)^8 J^6 + 3072 r^5 \sin(\theta)^6 J^4 + 384 r^9 \sin(\theta)^4 J^2 + 8 r^{13} \sin(\theta)^2) M^2 \\ & + (-16384 r^2 \sin(\theta)^8 J^6 - 1536 r^6 \sin(\theta)^6 J^4 + 2 r^{14} \sin(\theta)^2) M - 4096 r^3 \sin(\theta)^8 \\ & J^6 - 768 r^7 \sin(\theta)^6 J^4 - 48 r^{11} \sin(\theta)^4 J^2 - r^{15} \sin(\theta)^2 \big) \end{aligned}$$

(%i48)