

266 (4): No - Relativistic Quantization in 2 Theory
Consider the non relativistic Hamiltonian:

$$H = \frac{1}{2} m v^2 + U(r) \quad - (1)$$

In plane polar coordinates:

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad - (2)$$

and the force law is:

$$\begin{aligned} F(r) &= -\frac{L^2}{mr^3} \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \\ &= m \frac{d^2 r}{dt^2} - \frac{L^2}{mr^3} \quad - (3) \end{aligned}$$

Therefore the 1689 Leibniz equation is:

$$m \frac{d^2 r}{dt^2} = F(r) + \frac{L^2}{mr^3} = -\frac{L^2}{mr^3} \frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) \quad - (4)$$

For an elliptical orbit:

$$r = \frac{a}{1 + e \cos(\theta)} \quad - (5)$$

and

$$F(r) = -\frac{mMG}{r^2} \quad - (6)$$

The elliptical orbit however shows no additional structure or quantization.

2) Quantization appears with the precessing elliptical orbit:

$$r = \frac{d}{1 + e \cos(x\theta)} \quad - (7)$$

From eqs. (4) and (7):

$$\begin{aligned} m \frac{d^2 r}{dt^2} &= x^2 \left(\frac{L^2}{mr^3} - \frac{L^2}{mr^2 d} \right) \quad - (8) \\ &= x^2 \left(-\frac{mMG}{r^2} + \frac{L^2}{mr^3} \right) \end{aligned}$$

where

$$d = \frac{L^2}{m^2 MG} \quad - (9)$$

Note carefully that the right hand side of eq. (8) is x^2 times the right hand side of the 1689 Leibniz equation for an elliptical orbit.

The factor x for all planetary precessions

is

$$x = 1 + \frac{3MG}{c^2 d} \quad - (10)$$

From eqs. (3) and (8):

$$F(r) = m \frac{d^2 r}{dt^2} - \frac{L^2}{mr^3} \quad - (11)$$

s. the force law needed for quantization is:

$$F(r) = -x^2 \frac{mMg}{r^2} + (x^2 - 1) \frac{L^2}{mr^3} \quad - (12)$$

The potential needed for quantization is:

$$U(r) = - \int F(r) dr = -x^2 \frac{mMg}{r} + \frac{(x^2 - 1)L^2}{2mr^2} \quad - (13)$$

Therefore the Hamiltonian for quantization is:

$$H = \frac{1}{2}mv^2 - x^2 \frac{mMg}{r} + \frac{(x^2 - 1)L^2}{2mr^2} \quad - (14)$$

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Note carefully that quantization can be investigated by simply graphing Eq. (7).
Closed orbits are quantized orbits.

Atoms and Molecules

A precisely analogous procedure can be used in the simplest case of a hydrogen atom, "i.e. the Hamiltonian is:

$$H = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r} \quad - (15)$$

4) Therefore:

$$nmg \rightarrow \frac{e^2}{4\pi\epsilon_0} \quad - (16)$$

Quantization appears through the precessing ellipse:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (17)$$

and the Hamiltonian:

$$H = \frac{1}{2}mv^2 - \frac{x^2 e^2}{4\pi\epsilon_0 r} + \frac{(x^2 - 1)L^2}{2mr^2} \quad - (18)$$

where m is the electron mass, $-e$ is the electron charge, and ϵ_0 the vacuum permittivity.

If it is assumed that gravitation and electromagnetism are equivalent, as indicated in eq. (16), then for electromagnetism:

$$\begin{aligned} x &= 1 + \frac{3e^2}{4\pi\epsilon_0 mc^2 d} \quad - (19) \\ &= 1 + \frac{3ng}{c^2 d} \end{aligned}$$

Here d is the right semi lat. of the orbital.
The fine structure constant is:

$$5) \quad d_f = \frac{e^2}{4\pi\hbar c \epsilon_0} = 0.007297351 \quad - (20)$$

so

$$\boxed{x = 1 + 3\hbar \frac{d_f}{mc}} \quad - (21)$$

In this model Planck quantization occurs through

$$x = 1 + \hbar \left(\frac{3d_f}{mc} \right) \quad - (22)$$

The conventional route to quantization is the well known Schrodinger equation, which is stated for eq. (15), with:

$$H = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} \quad - (23)$$

and

$$\underline{p}\phi = -i\hbar \underline{\nabla}\phi \quad - (24)$$

s.

$$-\frac{\hbar^2}{2m} \nabla^2 \phi - \frac{e^2}{4\pi\epsilon_0 r} \phi = E\phi \quad - (25)$$

i.e.

$$\hat{H}\phi = E\phi \quad - (26)$$

It is very much easier to solve eq.

(17) then solve Eq. (25).

In order to introduce the l quantum number it is possible to use:

$$L^2 \psi = \hbar^2 l(l+1) \psi \quad (27)$$

in eq. (18). The conventional approach to the Schrodinger quantization uses:

$$H = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\epsilon_0 r} \quad (28)$$

$$= \frac{1}{2} m \left(\frac{d^2 r}{dt^2} + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) - \frac{e^2}{4\pi\epsilon_0 r}$$

where

$$\frac{d\theta}{dt} = \frac{L}{mr^2} \quad (29)$$

So:

$$H = \frac{1}{2} m \frac{d^2 r}{dt^2} + \frac{L^2}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r} \quad (30)$$

with the potential:

$$U = \frac{L^2}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r} \quad (31)$$
$$= \frac{\hbar^2 l(l+1)}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

Eq. (27) is actually a classical result of spherical harmonics.

) In the new route to quantization the principal quantum numbers are defined by:

$$n = x = 1 + \frac{3\hbar}{mc} \frac{d_f}{d} \quad - (32)$$

here the Compton wavelength of an electron is:

$$\lambda_c = \frac{h}{mc} = 2.426309 \times 10^{-12} \text{ m} \quad - (33)$$

and the fine structure constant is:

$$\alpha_f = \frac{e^2}{4\pi\hbar c\epsilon_0} = 0.007297351 \quad - (34)$$

Note that

$$\hbar = h / (2\pi) \quad - (35)$$

If

$$x = 1 + \frac{r_0}{d} \quad - (36)$$

then integer x occurs at:

$$d = \frac{r_0}{n-1} \quad - (37)$$

where

$$r_0 = \frac{3\hbar}{mc} d_f = \frac{3}{2\pi} \lambda_c d_f \quad - (38)$$

So

$$\alpha = \frac{3}{2\pi(n-1)} \lambda_c d_f \quad - (39)$$

For the quantum state:

8)

$$n = 1 - (40)$$

Then

$$d \rightarrow \infty \text{ and } \kappa \rightarrow 1 - (41)$$

and an ellipse is obtained. In the traditional quantization this is:

$$n = 1, \ell = 0 (1s) - (42)$$

From eqs. (36) and (37):

$$\kappa = 1 + n - 1 = n - (43)$$

For

$$\kappa = 2 - (44)$$

Eq. (17) develops structure, and in the traditional quantization:

$$\left. \begin{array}{l} n = 2, \ell = 0 (2s) \\ n = 2, \ell = 1 (2p) \end{array} \right\} - (45)$$

The S orbitals in the new quantization can be understood by using the circular limit of eq. (17):

$$d = r - (46)$$

s. that the ellipse becomes a circle. So eq. (37) is interpreted as:

$$\boxed{d = \frac{r_0}{n-1}, \quad n \neq 1} - (47)$$

This is a plausible theory, but it is

more systematic to regard the classical theory as a limit of the Bohr-Sommerfeld quantization, which leads to:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (48)$$

where

$$d = \frac{1}{K} = \frac{L^2}{mkZe^2} \left(1 + \frac{H}{mc^2} \right)^{-1} \quad - (49)$$

and

$$K = \frac{1}{4\pi\epsilon_0} \quad - (50)$$

for the H atom:

$$Z = 1 \quad - (51)$$

Here

$$x^2 = \frac{1}{\left(\frac{e^2}{4\pi\epsilon_0 c L} \right)^2} \quad - (52)$$

The principal quantum number is defined by:

$$\begin{aligned} n h &= \oint m \frac{dr}{dt} dr = \oint p_r dr \\ &= L \oint \left(\frac{1}{r} \frac{dr}{d\theta} \right)^2 d\theta \quad - (53) \end{aligned}$$

and the angular momentum quantum number l is:

$$\oint L d\theta = 2\pi L = l h \quad - (54)$$

The fine structure constant is:

$$\alpha_g = \frac{e^2}{4\pi\hbar c \epsilon_0} \quad - (55)$$

So

$$\boxed{x^2 = 1 - \left(\frac{\hbar \alpha_g}{L}\right)^2} \quad - (56)$$

So for

x , eq. (54) must be used:

$$L = \hbar \ell \quad - (57)$$

and

$$x^2 = 1 - \left(\frac{\alpha_g}{\ell}\right)^2 \quad - (58)$$

Summary

In the classical limit of the Sommerfeld Hamiltonian

$$T = (\gamma^2 - 1) m c^2 \rightarrow \frac{1}{2} m v^2 \quad - (59)$$

and

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (60)$$

where

$$x^2 = 1 - \left(\frac{\alpha_g}{\ell}\right)^2 \quad - (61)$$

and

$$d = \frac{L^2}{4\pi \epsilon_0 m e^2} \left(1 + \frac{H}{m c^2}\right)^{-1} \quad - (62)$$

The eccentricity ϵ may be varied