

279(7): Determination of Photon Mass from Reflection and Refraction, or Photon Theory

The two relevant equations are:

$$\omega = \omega_1 + \omega_2 \quad - (1)$$

and

$$\underline{k} = \underline{k}_1 + \underline{k}_2 \quad - (2)$$

is the notation of previous notes and papers. Therefore

$$\omega_2 = \omega - \omega_1 \quad - (3)$$

and

$$\underline{k}_2 = \underline{k} - \underline{k}_1 \quad - (4)$$

It follows that:

$$\omega_2^2 = \omega^2 + \omega_1^2 - 2\omega\omega_1 \quad - (5)$$

and

$$k_2^2 = k^2 + k_1^2 - 2kk_1 \cos \theta_3 \quad - (6)$$

where  $\theta_3$  is the angle between the incident and reflected vectors. So:

$$\omega_2^2 - c^2 k_2^2 = \omega^2 - c^2 k^2 + \omega_1^2 - c^2 k_1^2 - 2(\omega\omega_1 - c^2 k k_1 \cos \theta_3) \quad - (7)$$

Consider a photon of mass  $m$ , then it obeys the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad - (8)$$

where

$$E = \hbar \omega, \quad p = \hbar \underline{k} \quad - (9)$$

So :

$$\omega_2^2 - c^2 k_2^2 = \omega_1^2 - c^2 k_1^2 = \omega^2 - c^2 k^2 \\ = \left( \frac{hc}{\lambda} \right)^2 =: x \quad - (10)$$

From eqs. (7) and (10):

$$2(\omega\omega_1 - c^2 k k_1 \cos \theta_3) = x \quad - (11)$$

So

$$2c^2 k k_1 \cos \theta_3 = 2\omega\omega_1 - x \quad - (12)$$

and

$$4c^2 k^2 k_1^2 \cos^2 \theta_3 = (2\omega\omega_1 - x)^2 \quad - (13)$$

where:

$$k^2 = \frac{\omega^2}{c^2} - \left( \frac{hc}{\lambda} \right)^2 \quad - (14)$$

and

$$k_1^2 = \frac{\omega_1^2}{c^2} - \left( \frac{hc}{\lambda} \right)^2 \quad - (15)$$

It follows that:

$$4(\omega^2 - x)(\omega_1^2 - x) \cos^2 \theta_3 = (2\omega\omega_1 - x)^2 \quad - (16)$$

This is an equation for photon's ii term of  
 $\omega$  and  $\omega_1$ .

From eq. (16):

$$4 \cos^2 \theta_3 \left( \omega^2 \omega_1^2 - x(\omega^2 + \omega_1^2) + x^2 \right) = 4\omega^2 \omega_1^2 - 4\omega\omega_1 x + x^2 \quad - (17)$$

3) So:

$$x^2 (4 \cos^2 \theta_3 - 1) - 4(\omega^2 + \omega_1^2) x \cos^2 \theta_3 = \omega^2 \omega_1^2 (1 - 4 \cos^2 \theta_3) \quad - (18)$$

i.e.

$$(4 \cos^2 \theta_3 - 1) x^2 + (4 \cos^2 \theta_3 - 1) \omega^2 \omega_1^2 - 4(\omega^2 + \omega_1^2) x \cos^2 \theta_3 = 0 \quad - (19)$$

$$x^2 + \omega^2 \omega_1^2 - \frac{4(\omega^2 + \omega_1^2) x \cos^2 \theta_3}{4 \cos^2 \theta_3 - 1} = 0 \quad - (20)$$

Eq. (20) can be denoted:

$$x^2 + Bx + C = 0 \quad - (21)$$

where

$$B = \frac{4(\omega^2 + \omega_1^2) \cos^2 \theta_3}{1 - 4 \cos^2 \theta_3} \quad - (22)$$

and

$$C = \omega^2 \omega_1^2 \quad - (23)$$

Therefore:

$$x = \left( \frac{mc}{h} \right)^2 = \frac{1}{2} \left( -B \pm (B^2 - 4C)^{1/2} \right) \geq 0 \quad - (24)$$

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