

319(4) : Simplified Calculation of Light Deflection due to Gravitation

The orbit of a photon around the sun is in general a precessing hypersola:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (1)$$

but to an excellent approximation:

$$x \sim 1 \quad - (2)$$

$$r = \frac{d}{1 + \epsilon \cos \theta} \quad - (3)$$

so:

with a very large eccentricity ϵ . Due to the quasi-Lorentz covariance of the field equations of ECE2 theory the orbit is governed by a quasi-Minkowski metric:

$$c^2 d\tau^2 = (c^2 - v^2) dt^2 \quad - (4)$$

where

$$v^2 = \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \quad - (5)$$

As in UFT 215 and UFT 216:

$$v^2 = \left(\frac{xL\epsilon}{dm} \right)^2 \sin^2(x\theta) + \left(\frac{L}{mr} \right)^2 \quad - (6)$$

where

$$L = mr^2 \frac{d\theta}{dt} \quad - (7)$$

Here:

$$\sin^2(\theta) = 1 - \frac{1}{e^2} \left(\frac{d}{r} - 1 \right)^2 \quad - (8)$$

So:

$$v^2 = \left(\frac{L}{md} \right)^2 \left[\frac{2x^2 d}{r} - x^2 (1 - e^2) + \frac{d^2}{r^2} (1 - x^2) \right] \quad - (9)$$

When

$$x = 1 \quad - (10)$$

$$v^2 = \left(\frac{L}{md} \right)^2 \left[\frac{2d}{r} - (1 - e^2) \right] \quad - (11)$$

The orbit (3) is started from:

$$\underline{F} = m \underline{g} = - \frac{m M G}{r^2} \underline{e}_r \quad - (12)$$

if

$$L^2 = m^2 M G d \quad - (13)$$

so

$$\begin{aligned} v^2 &= \frac{M G}{d} \left(\frac{2d}{r} - (1 - e^2) \right) \quad - (14) \\ &= M G \left(\frac{2}{r} - \frac{(1 - e^2)}{d} \right) \end{aligned}$$

At closest approach:

$$r = R_0 = \frac{d}{1 + e} \quad - (15)$$

and by definition the semi major axis is:

$$a = \frac{d}{1 - e^2} \quad - (16)$$

3) So
$$v^2 = mG \left(\frac{2}{r} - \frac{1}{a} \right) \quad - (17)$$

Also:
$$v^2 = mG \left(\frac{2}{r} + \frac{(\epsilon+1)(\epsilon-1)}{d} \right) \quad - (18)$$

$$= mG \left(\frac{2}{r} + \frac{(\epsilon-1)}{R_0} \right)$$

at closest approach. So
$$v^2 = mG \left(\frac{2}{R_0} + \frac{(\epsilon-1)}{R_0} \right) \quad - (19)$$

$$= \frac{mG}{R_0} (1 + \epsilon)$$

The angle of deflection at closest approach is:

$$2\phi = \frac{2}{\epsilon} \quad - (20)$$

where
$$\epsilon = \frac{R_0 v^2}{mG} - 1 \quad - (21)$$

$$\sim \frac{R_0 v^2}{mG}$$

if
$$R_0 v^2 \gg mG \quad - (22)$$

The approximation:

$$2\phi = \frac{2MG}{R \cdot v^2} \quad - (23)$$

The experimentally derived result is:

$$2\phi = \frac{4MG}{R \cdot c^2} \quad - (24)$$

to high precision, so:

$$v^2 = \frac{1}{2} c^2 \quad - (25)$$

which is the same result as that obtained in Note 319/3).

From eq. (4):

$$d\tau^2 = \left(1 - \frac{v^2}{c^2}\right) dt^2 \quad - (26)$$

an equation which defines the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{dt}{d\tau} \quad - (27)$$

From eqs. (25) and (27) the Lorentz factor is

$$\gamma = 0.5^{-1/2} = \frac{1}{\sqrt{2}} = 0.707 \quad - (28)$$

The photon mass is given by:

$$E_0 = \gamma m c^2 \quad - (29)$$

5) So

$$n = \frac{\hbar \omega}{\gamma c^2} = \left(\frac{\hbar}{\gamma c^2} \right) \omega \quad - (30)$$

here

$$\hbar = 1.05459 \times 10^{-34} \text{ Js} \quad - (31)$$

$$c = 2.997925 \times 10^8 \text{ m s}^{-1}$$

$$\text{So } n = \left(\frac{\sqrt{2} \times 1.05459 \times 10^{-42}}{2.99792^2} \right) \omega \quad - (32)$$

$$= 1.66 \times 10^{-43} \omega \text{ kg}$$

This result is stated for a one photon beam of light grazing the sun.

If the average energy of an oscillator is used:

$$\langle \hbar \omega \rangle = \frac{\hbar \omega}{e^{(\hbar \omega / kT)} - 1} \quad - (33)$$

$$\text{So } n = \frac{\hbar \omega}{\gamma c^2} \left(\frac{1}{e^{(\hbar \omega / kT)} - 1} \right) \quad - (34)$$