

37(7): Quantized Energy for the Aharonov Bohm Spacetime  
 The Aharonov Bohm (AB) spacetime contains the  $W^\mu$   
 potential of ECE2, defined by:

$$W^\mu = W^{(0)} \Omega^\mu \quad - (1)$$

here  $W^{(0)} = \hbar / e$   $- (2)$   
 the quantum of magnetic flux and  $\Omega^\mu$  is the spin  
 connection four vector:

$$\Omega^\mu = (\Omega^0, \underline{\Omega}) \quad - (3)$$

The energy / momentum imparted to material matter from  
 the AB spacetime is defined by the minimal prescription:

$$p^\mu \rightarrow p^\mu + e W^\mu \quad - (4)$$

where a positive sign has been used to be consistent with  
 eq. (2). Therefore the overall process is:

$$p^\mu \rightarrow p^\mu + p_{AB}^\mu \quad - (5)$$

where  $p_{AB}^\mu = e W^{(0)} = \hbar (\Omega^0, \underline{\Omega}) \quad - (6)$   
 is the energy / momentum of the Aharonov Bohm spacetime.

So  $E_{AB} = \hbar c \Omega^0 \quad - (7)$

and  $\underline{p}_{AB} = \hbar \underline{\Omega} \quad - (8)$

and define:  $\omega_{AB} = c \Omega^0$ ;  $\underline{k}_{AB} = \underline{\Omega} \quad - (9)$

2) where  $\omega_{AB}$  is the angular frequency of the AB spacetime and  $\hbar_{AB}$  its wave vector.

Therefore the spicination for vector define the energy momentum for vector of the AB spacetime.

The de Broglie Einstein equations of the AB spacetime are:

$$E_{AB} = \hbar \omega_{AB} = \gamma m c^2 \quad (10)$$

$$\underline{p}_{AB} = \hbar \underline{k}_{AB} = \gamma m \underline{v}_{0AB} \quad (11)$$

where 
$$\gamma = \left( 1 - \frac{v_{0AB}^2}{c^2} \right)^{-1/2} \quad (12)$$

Here  $m$  is the mass of a particle associated with the AB spacetime, and  $\underline{v}_{0AB}$  is its velocity. Therefore the AB spacetime can be thought as an ensemble of relativistic particles  $m_{AB}$ .

The AB spacetime is quantized according to:

$$\underline{p}_{AB} \phi_{AB} = \hbar \underline{k}_{AB} \phi_{AB} = -i \hbar \underline{\nabla} \phi_{AB} \quad (13)$$

where  $\phi_{AB}$  is the wavefunction of the AB spacetime, and

$$E_{AB} \phi_{AB} = i \hbar \frac{\partial \phi_{AB}}{\partial t} \quad (14)$$

The AB wavefunction  $\phi_{AB}$  obeys the relativistic

3) wave equation :

$$(\square + \kappa_{AB}^2) \psi_{AB} = 0 \quad - (15)$$

where

$$\kappa_{AB} = \frac{mc}{\hbar} \quad - (16)$$

and

$$\psi_{AB} = \exp(-i(\omega_{AB} t - \underline{\kappa}_{AB} \cdot \underline{r}))$$

Eq. (15) is the limit of the ECE wave equation of the AB spacetime:

$$\square \psi_{AB} = R \psi_{AB} \quad - (18)$$

where

$$R = -\kappa_{AB}^2 \quad - (19)$$

Eq (15) is the quantized version of the Einstein energy equation of the AB spacetime:

$$E_{AB}^2 = c^2 p_{AB}^2 + m_{AB}^2 c^4 \quad - (20)$$

Therefore the AB spacetime is understood as a relativistic particle of mass  $m_{AB}$ . The process of taking energy and momentum from the AB spacetime is :

$$E \rightarrow E + E_{AB} \quad - (21)$$

$$\underline{p} \rightarrow \underline{p} + \underline{p}_{AB} \quad - (22)$$