

378(4): Development of the Theory of Precession with the Hamiltonian

1) Classical Hamiltonian

$$H = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{m\mu G}{(x^2 + y^2)^{1/2}} \quad - (1)$$

and the field equations give:

$$\frac{\ddot{x}}{\ddot{y}} = \frac{\kappa_x}{\kappa_y} \quad - (2)$$

The force equations are:

$$\ddot{x} = - \frac{m\mu G x}{(x^2 + y^2)^{3/2}} \quad - (3)$$

$$\ddot{y} = - \frac{m\mu G y}{(x^2 + y^2)^{3/2}} \quad - (4)$$

So

$$\frac{\ddot{x}}{\ddot{y}} = \frac{x}{y} = \frac{\kappa_x}{\kappa_y} \quad - (5)$$

and

$$x\kappa_y = y\kappa_x \quad - (6)$$

Differentiating both sides of Eq. (6) gives:

$$\dot{x}\kappa_y + x\dot{\kappa}_y = \dot{y}\kappa_x + y\dot{\kappa}_x \quad - (7)$$

The initial conditions for solving eqs. (3) and (4) are therefore:

$$\frac{\ddot{x}(0)}{\ddot{y}(0)} = \frac{x(0)}{y(0)} = \frac{\kappa_x(0)}{\kappa_y(0)} \quad - (8)$$

and:

$$2) \dot{x}(0)K_y(0) + x(0)\dot{K}_y(0) = \dot{y}(0)K_x(0) + y(0)\dot{K}_x(0) - (9)$$

So eqns. (3) and (4) are solved with these initial conditions

By inputting $K_x(0), K_y(0)$ as constants, we have:

$$\dot{K}_x(0) = \dot{K}_y(0) - (10)$$

and the initial conditions simplify. In general, K_x, K_y and K_y are not constants. The orbit is completely determined by integrating eqs. (3) and (4) numerically, so $x, y, \dot{x}, \dot{y}, x$ and y are known. Therefore:

$$xK_y = yK_x - (11)$$

$$\ddot{x}K_y = \ddot{y}K_x - (12)$$

$$\dot{x}K_y + x\dot{K}_y = \dot{y}K_x + y\dot{K}_x - (13)$$

It is also known that:

$$K_x = -\frac{x}{x^2 + y^2} - (14)$$

$$K_y = -\frac{y}{x^2 + y^2} - (15)$$

and

$$x = -\frac{K_x}{K_x^2 + K_y^2} - (16)$$

$$y = -\frac{K_y}{K_x^2 + K_y^2} - (17)$$

The initial orbital velocity is:

$$v^2(0) = \dot{x}(0)^2 + \dot{y}(0)^2 - (18)$$

3) and the initial position is:

$$r^2(0) = x^2(0) + y^2(0) \quad - (19)$$

and

$$H = \frac{1}{2} m v^2(0) - \frac{m M G}{r(0)} \quad - (20)$$

and this remains a constant of motion. From eqs. (16) and (17):

$$x^2 + y^2 = \frac{\dot{x}^2 + \dot{y}^2}{(\ddot{x}^2 + \ddot{y}^2)^2} = \frac{1}{\ddot{x}^2 + \ddot{y}^2} \quad - (21)$$

So:

$$r^2(0) = x^2(0) + y^2(0) = \frac{1}{\ddot{x}^2(0) + \ddot{y}^2(0)} \quad - (22)$$

Therefore:

$$H = \frac{1}{2} m v^2(0) - m M G (\ddot{x}^2(0) + \ddot{y}^2(0))^{1/2} \quad - (23)$$

and

$$v^2(0) = \frac{2}{m} \left(H + m M G (\ddot{x}^2(0) + \ddot{y}^2(0))^{1/2} \right)$$

$$= \dot{x}^2(0) + \dot{y}^2(0) \quad - (24)$$

It is clear from Eq. (22) that using $\ddot{x}(0)$ and $\ddot{y}(0)$ as input parameters will result in a different initial $r(0)$ and $v(0)$. This is an example of other expressions.

The initial velocity and position for the Newtonian ellipse are defined by:

$$v^2(0) = \frac{2}{r(0)} - \frac{1}{a} \quad (25)$$

where a is the semi major axis:

$$a = \frac{d}{1-e^2} \quad (26)$$

Therefore the Hamiltonian is, initially:

$$H = \frac{1}{2} m v^2(0) - \frac{mM}{r(0)} = -\frac{mM}{a} \quad (27)$$

From eqs. (24) and (27):

$$\begin{aligned} v^2(0) &= \frac{2}{m} \left(-\frac{mM}{a} + mM \left(\dot{x}^2(0) + \dot{y}^2(0) \right)^{1/2} \right) \\ &= 2M \left(\left(\dot{x}^2(0) + \dot{y}^2(0) \right)^{1/2} - \frac{1}{a} \right) \quad (28) \\ &= \dot{x}^2(0) + \dot{y}^2(0) \end{aligned}$$

Retrograde Precession

In this case the force equations are:

$$\ddot{x} = -\frac{M}{r^3} \frac{x}{\left(x^2 + y^2 \right)^{1/2}} \quad (29)$$

$$\ddot{y} = -\frac{M}{r^3} \frac{y}{\left(x^2 + y^2 \right)^{1/2}} \quad (30)$$

where:

$$\gamma = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad - (31)$$

Here v_0 is the Newtonian velocity defined by:

$$v_0^2 = MG \left(\frac{2}{r} - \frac{1}{a} \right) \quad - (32)$$

The Hamiltonian for retrograde precession is defined by:

$$H = \gamma mc^2 - \frac{mMG}{(x^2 + y^2)^{1/2}} \quad - (33)$$

So

$$\gamma = \frac{H}{mc^2} + \frac{MG}{c^2 (x^2 + y^2)^{1/2}} \quad - (34)$$

From eqs. (3), (4), (29) and (30) it is seen that the retrograde precession becomes more and more pronounced as:

$$\gamma \rightarrow \infty \quad - (35)$$

i.e. as

$$\frac{1}{\gamma^2} \rightarrow 0 \quad - (36)$$

This means

$$\left(1 - \frac{v_0^2}{c^2}\right)^{3/2} \rightarrow 0 \quad - (37)$$

i.e.

$$v_0 \rightarrow c \quad - (38)$$

If the orbit is regarded as approximately

b) Newtonian is the first approximation to the initial velocity is maximized from eq. (28):

$$K_x^2(0) + K_y^2(0) \rightarrow \infty \quad (31)$$

so a very large precession can be engineered by eq. (31).

In fact there is an upper bound on v_0^2 in ECE2 theory:

$$v_0^2 \rightarrow \frac{c^2}{2} \quad (32)$$

which is the condition for light deflection due to gravity (see earlier WFT papers). So:

$$2mG \left((K_x^2(0) + K_y^2(0))^{1/2} - \frac{1}{a} \right) \rightarrow \frac{c^2}{2} \quad (33)$$

Forward Precession

The force equations are:

$$\ddot{x} = \frac{mG}{\gamma (x^2 + y^2)^{3/2}} \left(\frac{\dot{x}\dot{y}y + x\dot{x}^2}{c^2} - x \right) \quad (34)$$

$$\ddot{y} = \frac{mG}{\gamma (x^2 + y^2)^{3/2}} \left(\frac{\dot{y}\dot{x}x + y\dot{y}^2}{c^2} - y \right) \quad (35)$$

and the difference between the equations and eqs. (3) and (4) again maximized when

$$\gamma \rightarrow 0 \quad (36)$$

and under condition (33).