

9) : Triple Check, Recalculation of 393(7)

Carried:

$$\begin{aligned}
 & \langle (\underline{r} + \underline{\delta r})(2\underline{r} \cdot \underline{\delta r} + \underline{\delta r} \cdot \underline{\delta r}) \rangle \\
 &= \langle \underline{i} (x + \delta x)(2x\delta x + 2y\delta y + 2z\delta z + \delta x^2 + \delta y^2 + \delta z^2) \rangle - (1) \\
 & \quad + \dots \\
 &= \underline{r} \langle \underline{\delta r} \cdot \underline{\delta r} \rangle \\
 & \quad + 2\underline{i} \left( \langle x^2 \delta x + xy\delta y + xz\delta z \rangle \right. \\
 & \quad \left. + \langle x(\delta x)^2 + y\delta x\delta y + z\delta x\delta z \rangle \right) \\
 & \quad + \dots \\
 &= \underline{r} \langle \underline{\delta r} \cdot \underline{\delta r} \rangle + 2x\underline{i} \langle (\delta x)^2 \rangle + 2y\underline{j} \langle (\delta y)^2 \rangle \\
 & \quad + 2z\underline{k} \langle (\delta z)^2 \rangle
 \end{aligned}$$

Now use:

$$\begin{aligned}
 \langle (\delta x)^2 \rangle &= \langle (\delta y)^2 \rangle = \langle (\delta z)^2 \rangle - (2) \\
 &= \frac{1}{3} \langle \underline{\delta r} \cdot \underline{\delta r} \rangle
 \end{aligned}$$

So:

$$\begin{aligned}
 & \langle (\underline{r} + \underline{\delta r})(2\underline{r} \cdot \underline{\delta r} + \underline{\delta r} \cdot \underline{\delta r}) \rangle - (3) \\
 &= \sum_3 \underline{r} \langle \underline{\delta r} \cdot \underline{\delta r} \rangle \quad \checkmark \checkmark \checkmark \quad \underline{\text{Q.E.D.}}
 \end{aligned}$$

The isotropy rules are:

$$\begin{aligned}
 \langle \delta y \delta z \rangle &= \langle \delta x \delta z \rangle = \langle (\delta x)^3 \rangle = 0 - (4) \\
 &\text{etc.}
 \end{aligned}$$