ECE2: EXACT DESCRIPTION OF LIGHT DEFLECTION DUE TO GRAVITATION AND PLANAR ORBITAL PRECESSION.

by

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ABSTRACT

ECE2 relativity is used straightforwardly to give an exact analytical description both of light deflection due to gravitation and precession of planar orbits. Therefore these precisely measured phenomena, known experimentally with great accuracy, are described with Cartan geometry in a space with finite curvature and torsion. ECE2 unifies the concepts of special and general relativity.

Keywords: ECE2, light deflection due to gravitation, precession of planar orbits.

UFT342

1. INTRODUCTION

In recent papers of this series {1 - 12}, notably UFT325, light deflection due to gravitation has been explained straightforwardly using the relativistic velocity to correct the Newtonian result. This method gives the precisely correct experimental result and imposes an upper bound on the Lorentz factor. One of the important conclusions is that massive particles can travel at the speed of light c. This fact is well known experimentally, for example electrons can be accelerated to the speed of light. The obsolete theory of special relativity prohibits a massive particle from reaching c. In ECE2 relativity therefore, photons with mass can travel at c, thus changing the entire structure of the standard model. In UFT328, planar orbital precession was explained qualitatively by numerical solution of the ECE2 hamiltonian and lagrangian regarded as simultaneous equations. In this paper both phenomena are described analytically with ECE2 theory in a straightforward way.

This paper is a synopsis of detailed calculations contained in the background notes posted with UFT342 on www.aias.us. To understand the paper, it is essential to read the background notes. Note 342(1) is description of mass and acceleration due to gravity with ECE2 gravitons, which are massive spin one bosons. In so doing concepts can be transferred directly from classical electro-statics to classical gravitation, because the field equations of ECE2 have the same structure for electrodynamics and gravitation. UFT342(2) gives the Newtonian law of gravitation for an object of finite dimensions in preparation for a graviton description of mass. UFT342(3) reviews the results of UFT325 and calculates the number of gravitons in the sun, thereby giving the mass of the graviton in this simple first theory. UFT324(4) and UFT324(5) give complete details of the calculation of the orbital velocity in the Newtonian theory. This calculation is used as the basis for the calculation of the relativistic velocity and the deflection of light due to gravitation in ECE2 (see UFT325).

Some preliminary calculations in UFT342(5) to UFT342(7) are corrected by computer algebra. Notes UFT324(8) and UFT324(9) are the basis for the calculation of planar orbital precession in ECE2. Precession is calculated in exactly the same way as light deflection, using the fundamental definition of relativistic velocity. No other concept is used, and the results agree exactly with experimental data in both cases. By Ockham's Razor and the Baconian foundations of science, this theory is preferred to all predecessor theories.

In Section 2, foundational definitions are reviewed, and a summary description given of the calculation of planar orbital precession. In Section 3 the results are analysed numerically and graphically.

2. ORBITAL PRECESSION

The orbital precession of a point such as the perihelion is known experimentally to great precision. The experimental data for all precessions in the universe can be summarized empirically by:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad - (1)$$

where

$$x = 1 - \frac{3MG}{c^2 \alpha} \quad - (2)$$

in plane polar coordinates (r, θ). Here M is the mass of the attracting object, G is Newton's constant, c is the universal constant known as the vacuum speed of light, α is the half right latitude (semi latus rectum), and ϵ is the eccentricity. Eq. (1) is valid if and only if:

$$1 - x \rightarrow 0 \quad - (3)$$

and this is the case experimentally for all known orbital precessions, inside and outside the

solar system. There are criticisms of the experimental claim and methods, but in general the

foregoing is accepted.

In ECE2 relativity (special relativity in a space with finite torsion and curvature),

the orbit ($\underline{1}$) is considered to be due to the hamiltonian:

$$H = \gamma mc^2 + U \quad -(4)$$

and the lagrangian:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(5)$$

in which the Lorentz factor is:

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(6)$$

Here m is the mass of the object in orbit around M, U is the potential energy of gravitational

attraction between m and M, and v_N is the orbital Newtonian velocity. The lagrangian

analysis of papers such as UFT325 and UFT328 defines a constant of motion:

$$L = \gamma mr^2 \dot{\theta} \quad -(7)$$

known as the relativistic angular momentum. Here:

$$\dot{\theta} = \frac{d\theta}{dt} \quad -(8)$$

is the relativistic angular velocity, which is not a constant of motion.

The Newtonian and non precessing planar orbit is the conic section:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(9)$$

and is described by the non-relativistic or classical hamiltonian:

$$H = \frac{1}{2} m V_N^2 + U \qquad -(10)$$

and classical lagrangian:

$$\mathcal{L} = \frac{1}{2} m V_N^2 - U. \qquad -(11)$$

In this case the quantity:

$$L_o = m r^2 \dot{\theta}_o \qquad -(12)$$

is the classical angular momentum, a constant of non-relativistic motion. The classical angular velocity is:

$$\dot{\theta}_o = \frac{d\theta_o}{dt}. \qquad -(13)$$

The relativistic orbital velocity v is defined (see notes) as:

$$v^2 = \frac{L^2}{m^2 r^4} \left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right] - (14)$$

in terms of the constant relativistic angular momentum L. From Eqs. (1) and (14):

$$v^2 = \frac{L^2}{m^2} \left(\frac{1}{r^2} + \frac{x^2 \epsilon^2}{d^2} \sin^2(x(\theta)) \right) - (15)$$

By definition the relativistic velocity is:

$$\underline{v} = \gamma \underline{v}_N \qquad -(16)$$

where $\underline{v}_N$ is the Newtonian velocity. Eq. (16) can be rewritten as:

$$v^2 = \frac{v_N^2}{1 - v_N^2} \qquad -(17)$$

From Eq. (9):

$$V_N^2 = \frac{L_0^2}{m^2} \cdot \left(\frac{1}{r^2} + \frac{\epsilon^2}{\alpha^2} \sin^2\theta \right) \quad -(18)$$

in which L_0 is the constant non relativistic angular momentum. It follows that:

$$L^2 \left(\frac{1}{r^2} + x \frac{\epsilon^2}{\alpha^2} \sin^2(x\theta) \right) = \frac{L_0^2 \left(\frac{1}{r^2} + \frac{\epsilon^2}{\alpha^2} \sin^2\theta \right)}{1 - \left(\frac{L_0}{mrc} \right)^2 \left(\frac{1}{r^2} + \frac{\epsilon^2}{\alpha^2} \sin^2\theta \right)} \quad -(19)$$

which is a new, generally valid, constraint equation linking the precessing and relativistic orbit to the classical, non-precessing and Newtonian orbit.

The properties of the constraint equation (19) are analysed numerically and graphically in Section 3.

The half right latitude can be expressed as:

$$\alpha = a \left(1 - \epsilon^2 \right) \quad -(20)$$

where a is the semi major axis of a classical elliptical orbit. The classical angular momentum can be expressed as:

$$L_0^2 = m^2 M G \alpha . \quad -(21)$$

3. NUMERICAL AND GRAPHICAL ANALYSIS OF THE CONSTRAINT EQUATION

(Section by Dr. Horst Eckardt)

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3 Numerical and graphical analysis of the constraint equation

The constraint equation (19) is investigated by computer algebra in the following. For given geometrical parameters of an orbit, the equation contains the orbit coordinates r and θ. Therefore we can try to extract the orbit $r(\theta)$ from this equation. We have to multiply by the denominator and by r^4, leading to a quartic equation in r or, more precisely, to a quadratic equation in r^2, with two complicated solutions. Only one solution gives a continuous orbit. When inserting numerical values, we have to choose the angular momenta L and L_0 in such a way that they are compatible with the half right latitude α. In Fig. 1 such a solution is shown for $\alpha = 1$. Obviously the orbit increases over several rounds in an unphysical way. The polar plot (Fig. 2) shows that there is precession, but with additional nodal points in the rotation of the ellipse. The end region at the right hand side has been enlarged in Fig. 3. There is indeed precession. We had to use a small x factor of 1.005 because the constraint equation holds only for small deviations from an ellipse. So the precession is quite small everywhere.

As an alternative, we have assumed a Newtonian behaviour for the orbit at the right hand side of the constraint equation (because this part is derived from the Newtonian velocity). This is the orbit given by Eq.(9). Then the resulting orbit $r(\theta)$ is stable (see Fig. 4). The polar plot now has no additional nodal points (Fig. 5) and precession is again visible in an enlarged view (Fig. 6).

Next we have investigated the relativistic γ factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_N^2}{c^2}}} \tag{22}$$

with Newtonian frame velocity

$$v_N^2 = \frac{L_0^2}{m^2}\left(\frac{\epsilon^2 \sin(\theta)^2}{\alpha^2} + \frac{1}{r^2}\right). \tag{23}$$

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For the parameters chosen, γ varies between 1.015 and 1.06 which is a small range. One can try to estimate a precession effect of γ by inserting the velocity v derived from the x dependent orbit:

$$v^2 = \frac{L_0^2}{m^2} \left(\frac{\epsilon^2 \, x^2 \sin{(\theta \, x)}^2}{\alpha^2} + \frac{(\epsilon \cos{(\theta \, x)} + 1)^2}{\alpha^2} \right). \tag{24}$$

The result is graphed in Fig. 7. There is no visible deviation from a circle, therefore precession is also extremely small. Using $\gamma \approx const.$ may be a good approximation in certain cases.

The approach (1) for the precessing orbit can be improved by allowing for x to vary with θ. Then the derivative $dr/d\theta$ will have an additional term containing $dx/d\theta$:

$$\frac{dr}{d\theta} = \frac{1}{\alpha} \mathrm{r}\,(\theta)^2 \, \epsilon \, \left(\theta \, \frac{d\,\mathrm{x}\,(\theta)}{d\,\theta} + \mathrm{x}\,(\theta) \right) \, \sin{(\theta\,\mathrm{x}\,(\theta))} \tag{25}$$

and

$$v^2 = \frac{L^2}{m^2} \left(\frac{\epsilon^2 \left(\theta \, \frac{d\,\mathrm{x}(\theta)}{d\,\theta} + \mathrm{x}\,(\theta) \right)^2 \sin{(\theta\,\mathrm{x}\,(\theta))}^2}{\alpha^2} + \frac{1}{\mathrm{r}\,(\theta)^2} \right). \tag{26}$$

Inserting this into the left hand side of the constraint equation (19), we obtain a differential equation for $x(\theta)$. This equation is non-linear and not solvable analytically. As a simplification, we can neglect the derivative of $x(\theta)$ and resolve the constraint equation for x. This gives a transcendent equation because x appears additionally in the argument of the cosine function, so this equation is not analytically solvable too. However we can derive an expression for $x(\theta)$ in the following way:

From Eqs.(7) and (12) we obtain

$$\frac{L}{L_0} = \frac{\gamma \, m \, r^2 \, \dot{\theta}}{m \, r^2 \, \dot{\theta}_0} = \gamma \, \frac{\dot{\theta}}{\dot{\theta}_0}. \tag{27}$$

Comparing the orbits (1) and (9), the relation between θ and θ_0 is

$$\theta = x \, \theta_0. \tag{28}$$

Assuming a moderately varying x compared to the angular velocities, we have

$$\dot{\theta} \approx x \, \dot{\theta}_0, \tag{29}$$

therefore

$$\frac{L}{L_0} = \gamma \, x, \tag{30}$$

$$x = \frac{1}{\gamma} \frac{L}{L_0}. \tag{31}$$

With Eq.(22) for γ we then have a dependence of x on θ. The variation of x is significant, it is graphed in Fig. 8. Inserting this into (24), which is the

left hand side of the constraint equation, we obtain an expression for the orbit $r(\theta)$ (with using the non-relativistic orbit at the right hand side). The result is a precessing orbit again (Fig. 9) which can be compared with Fig. 5. The precession effect is much larger here because x varies between 0.975 and 1.02. Fig. 5 has much similarity to Fig. 1 of UFT paper 328 where the numerical solution for the orbit has been plotted. In particular the direction of precession is the same as in paper 328. Another point concluded from paper 328 is that the relativistic orbit - even without precession - differs significantly from the non-relativistic orbit. Therefore the constraint equation of this paper - where r appears on both sides of the equation - can only be an approximation for small precession effects. An exact treatment requires the numerical method of paper 328 where the relativistic Lagrangian and Hamiltonian have been solved simultaneously.
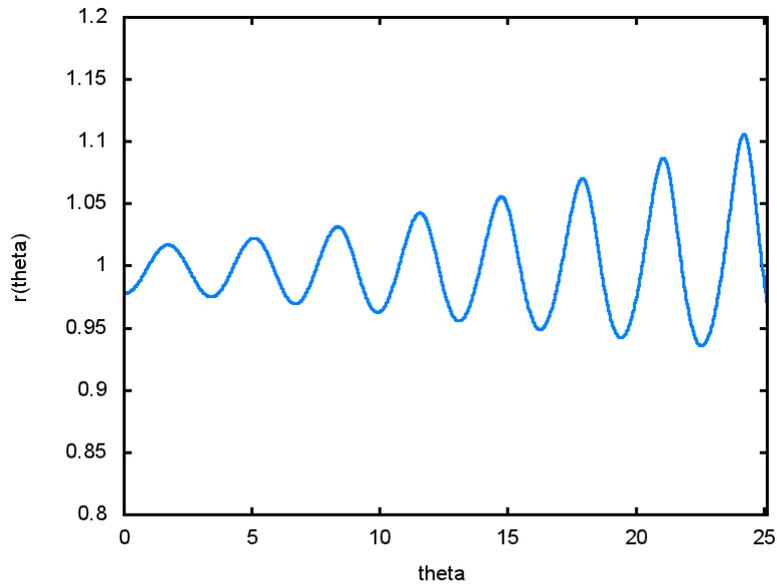
Figure 1: Orbit $r(\theta)$ from constraint equation.

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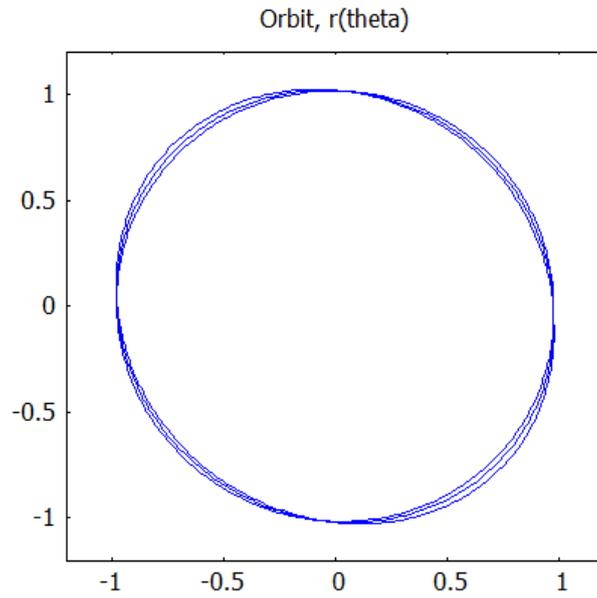
Figure 2: Polar plot of orbit $r(\theta)$ from constraint equation.
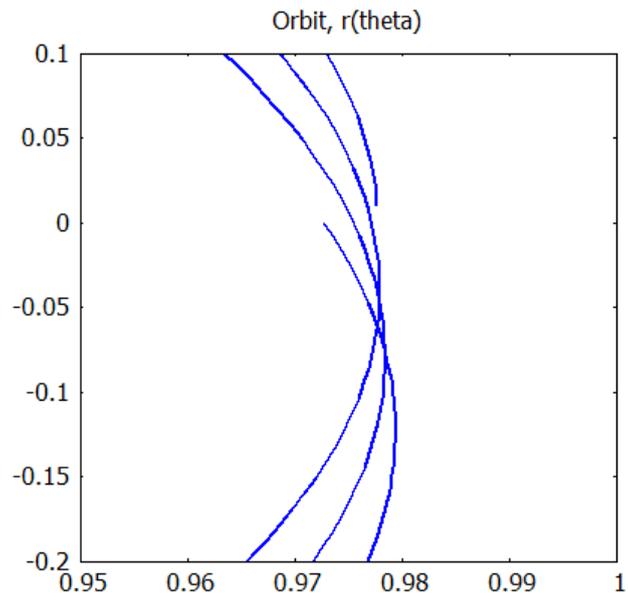
Figure 3: Polar plot of segment of orbit $r(\theta)$ from constraint equation.
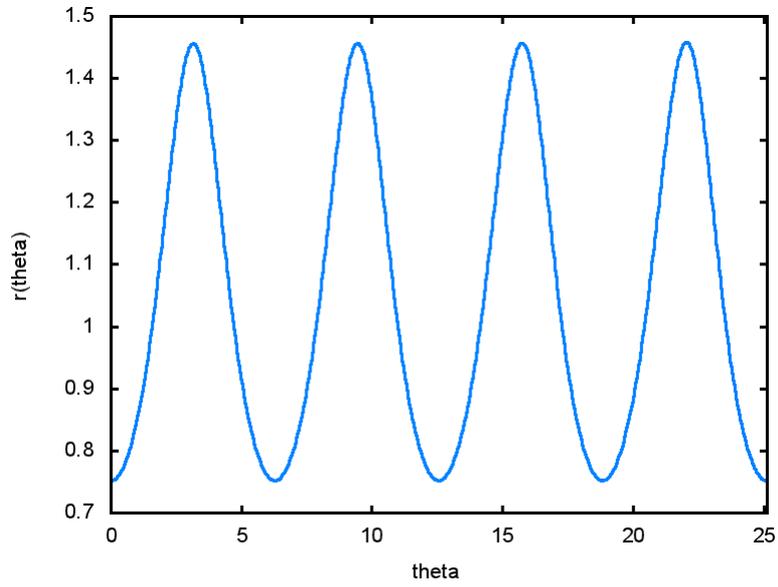
Figure 4: Orbit $r(\theta)$ from reduced constraint equation.
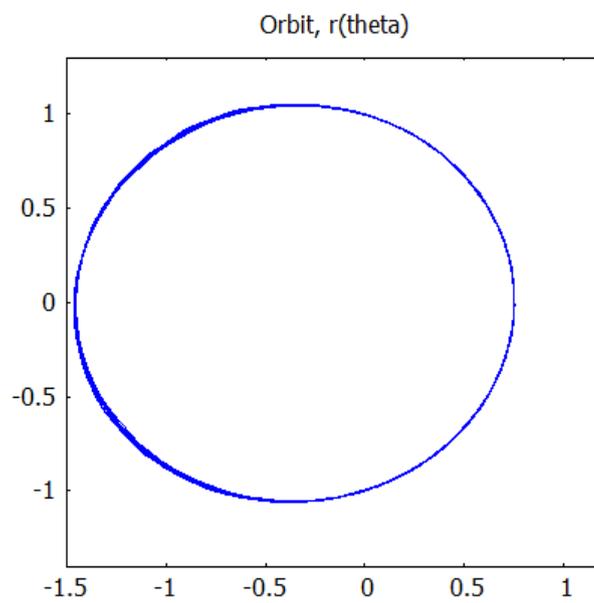
Figure 5: Polar plot of orbit $r(\theta)$ from reduced constraint equation.

5
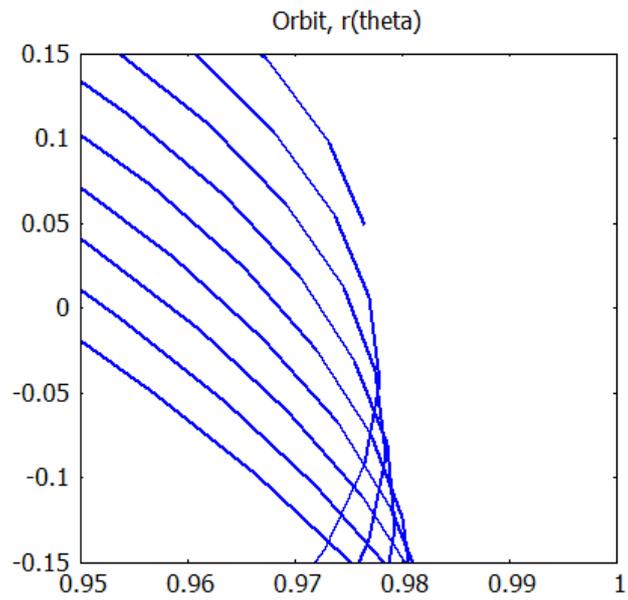
Figure 6: Polar plot of segment of orbit $r(\theta)$ from reduced constraint equation.
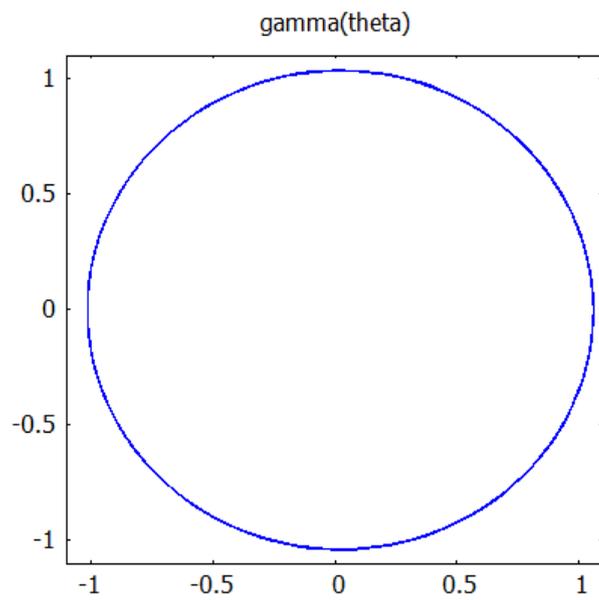
Figure 7: Polar plot of $\gamma(\theta)$.
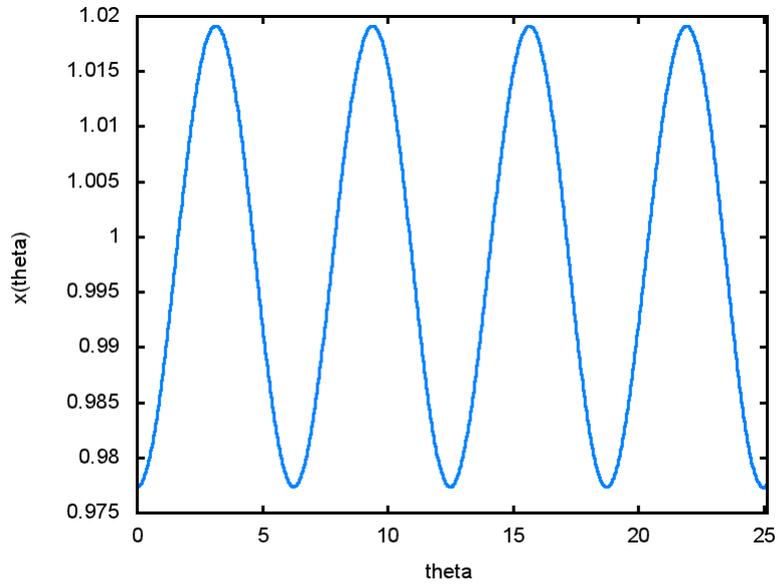
Figure 8: Variable $x(\theta)$ from ratio L/L_0.
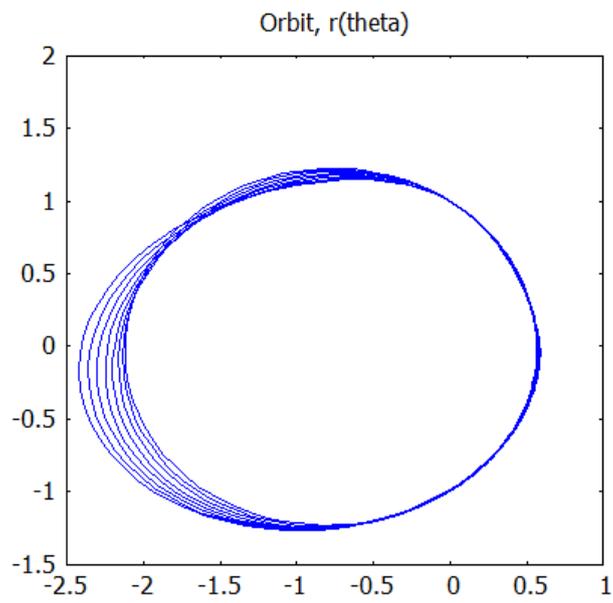
Figure 9: Polar plot of orbit $r(\theta)$ with variable $x(\theta)$.

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ACKNOWLEDGMENTS

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