# EXPLANATION OF ORBITAL PRECESSION WITH FLUID DYNAMICS

by

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ABSTRACT

The principles of fluid dynamics are applied to orbital dynamics and it is shown that a precise and simple explanation emerges of orbital precession to experimental precision. These methods generalize classical dynamics and produces new fundamental velocities and accelerations in the plane polar coordinate system, extending the work of Coriolis in 1835.

Keywords: ECE2, unification of fluid dynamics and dynamics, planetary precession.

UFT 363

## 1. INTRODUCTION

In immediately preceding papers of this series {1 - 12}, the ECE2 unification of fluid dynamics and gravitational theory has led to new fundamental inferences in cosmology. Spacetime is considered to be governed by the equations of fluid dynamics within the context of Cartan geometry, so the four derivative is replaced by the Cartan covariant derivative, an example of which is the well known convective derivative of fluid dynamics. The structure of the field equations of fluid dynamics has been shown to be the same as that of electrodynamics and gravitational theory. This is an example of the use of ECE2 unified field theory. The field equations are written in a mathematical space with finite torsion and curvature, and are Lorentz covariant. These properties have been described as ECE2 covariance.

This paper is a synopsis of detailed calculations and computer algebra in the notes accompanying UFT363 on www.aias.us and www.upitec.org, archived on www.archive.org. Notes 363(1) and 363(2) give a preliminary description of precession due to a fluid vacuum, using well defined approximations. Note 363(3) is a preliminary description without approximation, resulting in a novel force equation in which the effect of using a convective derivative is evaluated through spin connection components. Section 2 of this paper is based on Notes 363(4) and 363(5), in which Eulerian first principles in fluid dynamics are used to give a precise description of orbital precession. Section 3 is a summary of computer methods which could be applied to the development of the hamiltonian and lagrangian.

## 2. ORBITAL PRECESSION CALCULATION

Consider the fundamentals of Eulerian fluid dynamics, in which any vector field $\underline{F}$ is defined as a function of position, $\underline{r}$, and time t:

$$\underline{F} = \underline{F}(\underline{r}, t). \quad -(1)$$

In the plane polar coordinate system the vector field is defined by:

$$\underline{F} = F_r \underline{e}_r + F_\theta \underline{e}_\theta. \quad -(2)$$

It follows as in Note 362(5) of UFT362 that the convective derivative of $\underline{F}$ is:

$$\frac{D\underline{F}}{Dt} = \frac{\partial \underline{F}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{F}. \quad -(3)$$

In component notation Eq. ( 3 ) becomes:

$$\frac{D}{Dt}\begin{bmatrix} F_r \\ F_\theta \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} F_r \\ F_\theta \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} F_r \\ F_\theta \end{bmatrix} + \begin{bmatrix} \frac{\partial F_r}{\partial r} & \frac{1}{r}\frac{\partial F_r}{\partial \theta} \\ \frac{\partial F_\theta}{\partial r} & \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} \end{bmatrix}\begin{bmatrix} v_r \\ v_\theta \end{bmatrix}. \quad -(4)$$

The second term on the right hand side of this equation is the result of the rotation of the coordinate axes of the plane polar system. The third term is a consequence of the fact that $\underline{F}$ is a function of $\underline{r}$ as well as t. This is the usual assumption used in fluid dynamics, and in this paper is applied to orbital theory to give an exact explanation of orbital precession.

In classical dynamics:

$$\underline{F} = \underline{F}(t) \quad -(5)$$

and Eq. ( 4 ) reduces to:

$$\frac{D}{Dt}\begin{bmatrix} F_r \\ F_\theta \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} F_r \\ F_\theta \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} F_r \\ F_\theta \end{bmatrix}. \quad -(6)$$

If $\underline{F}$ is velocity or acceleration in Eq. ( 6 ) the well known Coriolis velocity and accelerations emerge. Therefore in classical orbital theory:

$$\begin{bmatrix} \frac{\partial F_r}{\partial r} & \frac{1}{r}\frac{\partial F_r}{\partial \theta} \\ \frac{\partial F_\theta}{\partial r} & \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad -(7)$$

In Eulerian fluid dynamics the right hand side of Eq. ( 7 ) is not zero

because:

$$\underline{F}(t) \rightarrow \underline{F}(\underline{r}, t). \quad -(8)$$

Consider the position vector of a fluid element, denoted by:

$$\underline{R} = \underline{R}(\underline{r}, t). \quad -(9)$$

It follows that the velocity field of the fluid is defined by:

$$\underline{v}(\underline{r}, t) = \frac{D\underline{R}}{Dt} = \frac{\partial \underline{R}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{R} \quad -(10)$$

or in component format in the plane polar coordinate system:

$$\frac{D}{Dt}\begin{bmatrix} R_r \\ R_\theta \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} R_r \\ R_\theta \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} R_r \\ R_\theta \end{bmatrix} + \begin{bmatrix} \frac{\partial R_r}{\partial r} & \frac{1}{r}\frac{\partial R_r}{\partial \theta} \\ \frac{\partial R_\theta}{\partial r} & \frac{1}{r}\frac{\partial R_\theta}{\partial \theta} \end{bmatrix}\begin{bmatrix} V_r \\ V_\theta \end{bmatrix}. \quad -(11)$$

In plane polar coordinates:

$$\underline{R} = R\,\underline{e}_r \quad -(12)$$

so:

$$R_r = R, \quad R_\theta = 0 \quad -(13)$$

The spin connection matrix needed to define the velocity field is therefore:

$$\begin{bmatrix} \Omega^1{}_{01} & \Omega^1{}_{02} \\ \Omega^2{}_{01} & \Omega^2{}_{02} \end{bmatrix} = \begin{bmatrix} \frac{\partial R_r}{\partial r} & \frac{1}{r}\frac{\partial R_r}{\partial \theta} \\ 0 & 0 \end{bmatrix} \quad -(14)$$

with components:

$$\Omega^1{}_{01} = \frac{\partial R_r}{\partial r}, \quad \Omega^1{}_{02} = \frac{1}{r}\frac{\partial R_r}{\partial \theta},$$
$$\Omega^2{}_{01} = 0, \quad \Omega^2{}_{02} = 0. \quad -(15)$$

The velocity field components are therefore:

$$v_r = \left(1 + \Omega^1{}_{01}\right)\dot{r} + \Omega^1{}_{02}\,\omega r \quad -(16)$$

and

$$v_\theta = \dot{\theta} r = \omega r. \quad -(17)$$

The hamiltonian of the system is:

$$H = \frac{1}{2}m\left(v_r^2 + v_\theta^2\right) + U \quad -(18)$$

and the lagrangian is:

$$\mathcal{L} = \frac{1}{2}m\left(v_r^2 + v_\theta^2\right) - U \quad -(19)$$

where U is the potential energy.

It is known that solar system precession is a very tiny effect, so:

$$\Omega^1{}_{01} \sim \Omega^1{}_{02} \ll 1 \quad -(20)$$

but in other systems such as spiral galaxies the spin connection components may change the orbit from the inverse square law to the observed inverse cube law of spiral galaxies. In classical dynamics:

$$\underline{v}(t) = \frac{D\underline{r}(t)}{Dt} = \frac{\partial \underline{r}(t)}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{r}(t) \quad -(21)$$

so the Coriolis velocity in plane polar coordinate sis given by the convective derivative of the position $\underline{r}(t)$ of a particle rather than $\underline{R}(\underline{r}, t)$ of a fluid element::

$$\begin{bmatrix} v_r \\ v_\theta \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} r(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} r(t) \\ 0 \end{bmatrix}. \quad -(22)$$

In component form, Eq. ( 22 ) is:

$$v_r = \frac{\partial r(t)}{\partial t} \quad -(23)$$

$$v_\theta = \dot{\theta} r(t) \quad -(24)$$

which gives the Coriolis velocity:

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \quad -(25)$$

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The Euler Lagrange equations of the system are:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(26)$$

and:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}}. \quad -(27)$$

From Eq. ( 26 ) a new force law can be found from the lagrangian:

$$\mathcal{L} = \frac{1}{2}m\left((1+\Omega'_{01})^2\dot{r}^2 + \Omega'^2_{02} r^2\dot{\theta}^2 + 2\Omega'_{02}(1+\Omega'_{01})\dot{r}\dot{\theta}r + \dot{\theta}^2 r^2\right) - U \quad -(28)$$

From Eq. ( 28 ):

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = m\left(\left(1+\Omega'_{01}\right)^2 \dot{r} + \Omega'_{02}\left(1+\Omega'_{01}\right)\dot{\theta}\, r\right) \quad -(29)$$

and:

$$\frac{\partial \mathcal{L}}{\partial r} = m\left(\left(1+\Omega'^{2}_{02}\right) r\dot{\theta}^2 + \Omega'_{02}\left(1+\Omega'_{01}\right)\dot{r}\dot{\theta}\right) - \frac{\partial U}{\partial r}. \quad -(30)$$

It follows that the force law of any planar orbit is:

$$F = m\left(\left(1+\Omega'_{01}\right)^2 \ddot{r} + \Omega'_{02}\left(1+\Omega'_{01}\right)\left(r\ddot{\theta} + \dot{\theta}\dot{r}\right) - \left(1+\Omega'^{2}_{02}\right) r\dot{\theta}^2 - \Omega'_{02}\left(1+\Omega'_{01}\right)\dot{r}\dot{\theta}\right). \quad -(31)$$

In comparison, the force law of a conic section orbit is well known to be:

$$F = m\left(\ddot{r} - r\dot{\theta}^2\right). \quad -(32)$$

So the orbit is changed by $\Omega'_{01}$ and $\Omega'_{02}$.

Assume that:

$$\Omega'_{02} = \frac{1}{r}\frac{\partial R_r}{\partial \theta} \sim 0 \quad -(33)$$

and the lagrangian simplifies to:

$$\mathcal{L} = \frac{1}{2} m\left(\left(1+\Omega'_{01}\right)^2 \dot{r}^2 + \dot{\theta}^2 r^2\right) - U. \quad -(34)$$

From the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(35)$$

and the angular momentum is:

$$L = \frac{\partial \mathcal{L}}{\partial \theta} = mr^2\theta \quad -(36)$$

From Eq. (35):

$$\frac{dL}{dt} = 0 \quad (37)$$

so the angular momentum is conserved.

In the approximation (33) the force law (31) becomes:

$$F = m\left(\left(1+\Omega^1_{01}\right)^2 \ddot{r} - r\dot{\theta}^2\right). \quad (38)$$

Using the Binet variable {1 - 12}:

$$u = \frac{1}{r} \quad (39)$$

it follows that:

$$\ddot{r} = -\frac{L^2}{m^2}u^2\frac{d^2u}{d\theta^2}, \quad r\dot{\theta}^2 = \frac{L^2}{m^2}u^3. \quad (40)$$

From Eqs. (38) and (40):

$$F = -\frac{L^2}{m}\left(\left(1+\Omega^1_{01}\right)^2 u^2\frac{d^2u}{d\theta^2} + u^3\right) \quad (41)$$

so the following generalized Binet equation is obtained:

$$\left(1+\Omega^1_{01}\right)^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2}F(r). \quad (42)$$

Eq. (42) can be applied to orbital theory and to fluid dynamics in the approximation (33) and reduces to the well known Binet equation of orbital theory in the limit:

$$\Omega^1_{01} \rightarrow 0. \quad (43)$$

For example, if the orbit is a conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(44)$$

where $\alpha$ is the half right latitude and $\epsilon$ is the eccentricity, it follows that:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\cos\theta \quad -(45)$$

and:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} = -\frac{mr^2}{L}F \quad -(46)$$

Therefore the force law of the orbit is:

$$F(r) = -L^2/(m\alpha r^2). \quad -(47)$$

From Newtonian orbital theory {1 - 12} it is well known that:

$$L^2 = m^2 MG\alpha \quad -(48)$$

where M is the mass of the gravitating object and G is Newton's constant, so:

$$F = -\frac{mMG}{r^2} \quad -(49)$$

which is the inverse square law, QED.

From Eq. (42) however:

$$-\frac{mr^2}{L^2}F(r) = -\frac{\epsilon}{\alpha}\left(1+\Omega'_{01}\right)^2\cos\theta + \frac{1}{\alpha}\left(1+\epsilon\cos\theta\right) \quad -(50)$$

From Eq. (44):

$$\frac{\epsilon}{\alpha}\cos\theta = \frac{1}{r} - \frac{1}{\alpha} \quad -(51)$$

so:

$$-\frac{mr^2}{L^2}F(r) = \frac{1}{\alpha} + \left(\frac{1}{r}-\frac{1}{\alpha}\right)\left(1-\left(1+\Omega'_{01}\right)^2\right)$$

$$= \frac{1}{\alpha}\left(1 - \left(1 - (1+\Omega'_{01})^2\right)\right) + \frac{1}{r}\left(1 - (1+\Omega'_{01})^2\right) \quad -(52)$$

Denote:

$$y = 1 - (1 + \Omega'_{01})^2 \quad -(53)$$

it follows that the force law is:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{1-y}{\alpha} + \frac{y}{r}\right) \quad -(54)$$

In UFT193 it was shown that the force law for a precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(55)$$

is:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{x^2}{\alpha} + \frac{1}{r}(1-x^2)\right) \quad -(56)$$

using the Binet equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2}F(r) \quad -(57)$$

So:

$$y = 1 - x^2 \quad -(58)$$

and:

$$x = 1 + \Omega'_{01} \quad -(59)$$

To contemporary experimental precision:

$$x = 1 + \frac{3MG}{\alpha c^2} \qquad (60)$$

where c is the vacuum speed of light. Therefore:

$$\Omega^1{}_{01} = \frac{\partial R_r}{\partial r} = \frac{3MG}{\alpha c^2}. \qquad (61)$$

Orbital precession is due to the effect of the fluid dynamic function $\partial R_r / \partial r$, which can be thought of as the rate of displacement of a position element of a fluid dynamic spacetime, aether or vacuum. This spin connection makes the orbit precess. The conic section orbit used in the generalized Binet equation ( 42 ) is exactly equivalent to the use of the precessing orbit ( 55 ) in the usual Binet equation ( 57 ). In both cases the law of attraction is a combination of inverse square and inverse cube in r. This is not the Einstein result, which is inverse square plus inverse fourth power in r. The Einstein theory does not in fact give a precessing orbit, as shown in previous UFT papers.

3. GRAPHICS OF PRECESSION AND THE NEW FORCE LAW.

(Section by Dr. Horst Eckardt)

# Explanation of orbital precession with fluid dynamics

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## 3 Graphics of precession and new force law

We compute the acceleration $\mathbf{a} = \mathbf{F}/m$ in the Newtonian limit (elliptic orbit). As described in section 2, then we have:

$$r = \frac{\alpha}{1 + \epsilon \cos(\theta)}, \tag{62}$$

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha} \sin(\theta), \tag{63}$$

$$\omega = \dot{\theta} = \frac{L}{mr^2}. \tag{64}$$

In general the acceleration has a radial and an angular component which in the case of a fluid dynamics spacetime is expressed by

$$\begin{aligned} \mathbf{a} = & \left( -\frac{MG}{r^2} + {\Omega^1}_{01} \frac{L}{mr^2} \frac{dr}{d\theta} + {\Omega^1}_{02} \frac{L}{mr} \right) \mathbf{e}_r \\ & + \left( {\Omega^2}_{01} \frac{L}{mr^2} \frac{dr}{d\theta} + {\Omega^2}_{02} \frac{L}{mr} \right) \mathbf{e}_\theta \end{aligned} \tag{65}$$

(see note 363(3)). We present graphical examples for the acceleration components. By using Eqs. (62 and 63), the components of (65) can be expressed either as a function of $r$ or a function of $\theta$. We present both possibilities in Figs. 1 and 2, blue and green lines. The Newtonian form can be obtained by setting all spin connection components to zero in Eq. (65). According to Newtonian theory, there is no angular component of acceleration as can be seen from from both figures.

For the second case, we use the full form (65) for acceleration with choice of

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spin connections:

$$\Omega^1{}_{01} = 0.2, \tag{66}$$

$$\Omega^1{}_{02} = -0.2, \tag{67}$$

$$\Omega^2{}_{01} = 0.2, \tag{68}$$

$$\Omega^2{}_{02} = -0.2. \tag{69}$$

In this non-Newtonian case, both acceleration components are different from zero, denoted by "P" in Figs. 1 and 2 (red and purple lines). These components (Fig. 1) are now defined only in the range of the elliptic orbit and a bit more negative than in the Newtonian case denote by "N". The angular component varies with angle $\theta$ as can seen from Fig. 2. The acceleration components are periodic in $2\pi$ as required.

So far we have assumed an elliptic orbit even for the non-Newtonian case. Actually it is a precessing ellipse as can be seen from the solution of the Lagrange equations obtained from the Lagrangian (34). We have assumed that only $\Omega^1{}_{01}$ is significantly different from zero. Then we obtain the equations of motion

$$\ddot{\theta} = -\frac{2\dot{r}\,\dot{\theta}}{r}, \tag{70}$$

$$\ddot{r} = \frac{r^3\,\dot{\theta}^2 - GM}{\left(\Omega^1{}_{01} + 1\right)^2\, r^2}, \tag{71}$$

which differ from the Newtonian form by the spin connection in the denominator of the second equation. These equations have been solved numerically by using initial conditions of bound orbits. This gives the trajectories $\theta(t)$ and $r(t)$ as graphed in Fig. 3. The three-dimensional orbit plot shows that the orbit is not closed but a precessing ellipse in the plane $Z = 0$. Obviously the existence of one fluid dynamic spin connection term suffices to result in non-Newtonian orbits. Alternatively, such precessing ellipses were obtained in UFT paper 328 by relativistic effects.

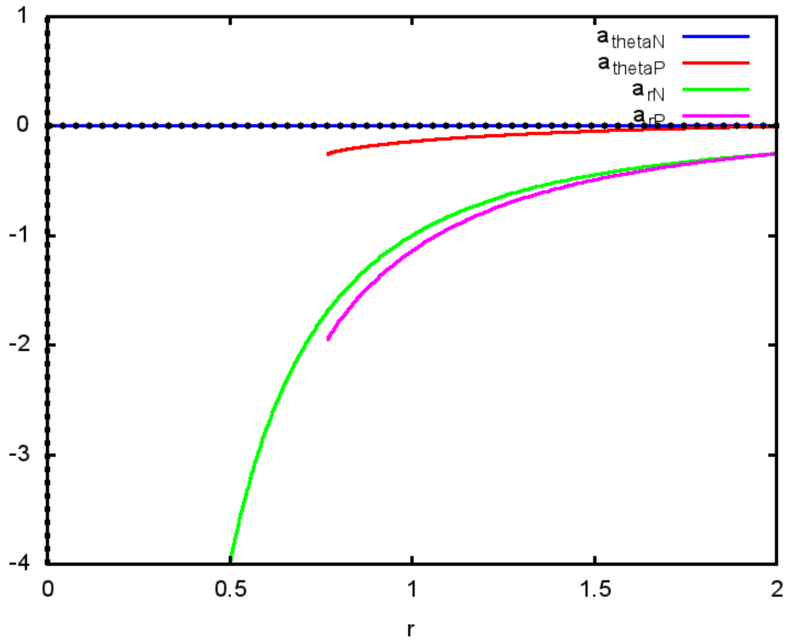


Figure 1: Acceleration components (Newtonian N and non-Newtonian P) in dependence of $r$.

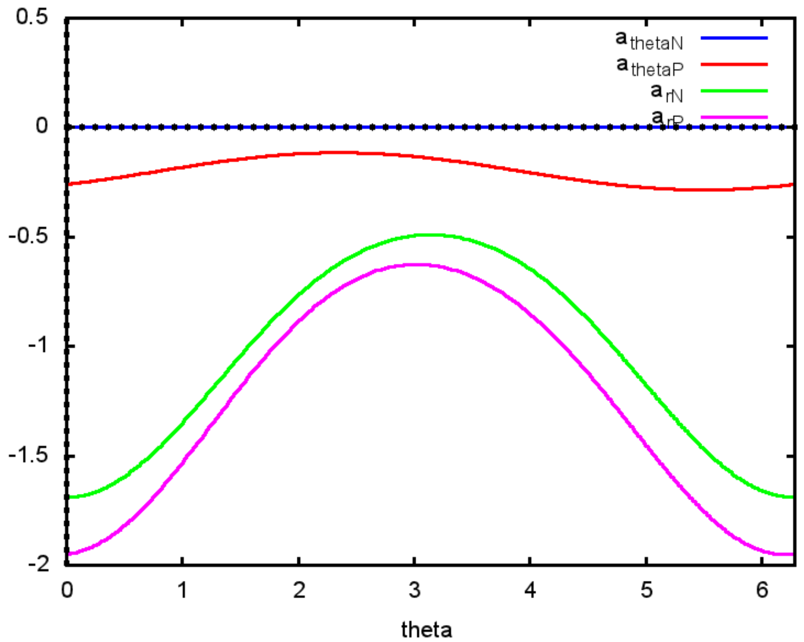


Figure 2: Acceleration components (Newtonian N and non-Newtonian P) in dependence of $\theta$.

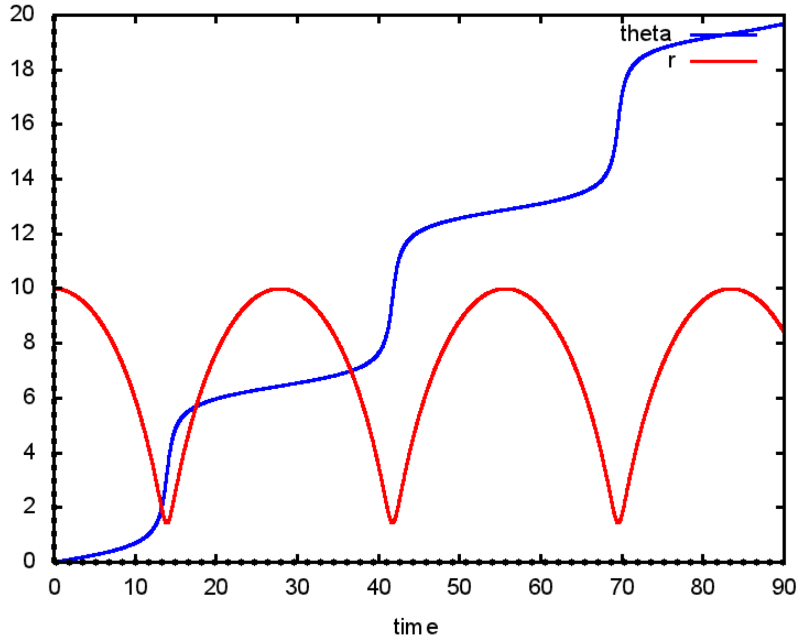


Figure 3: Trajectories $\theta(t)$ and $r(t)$.

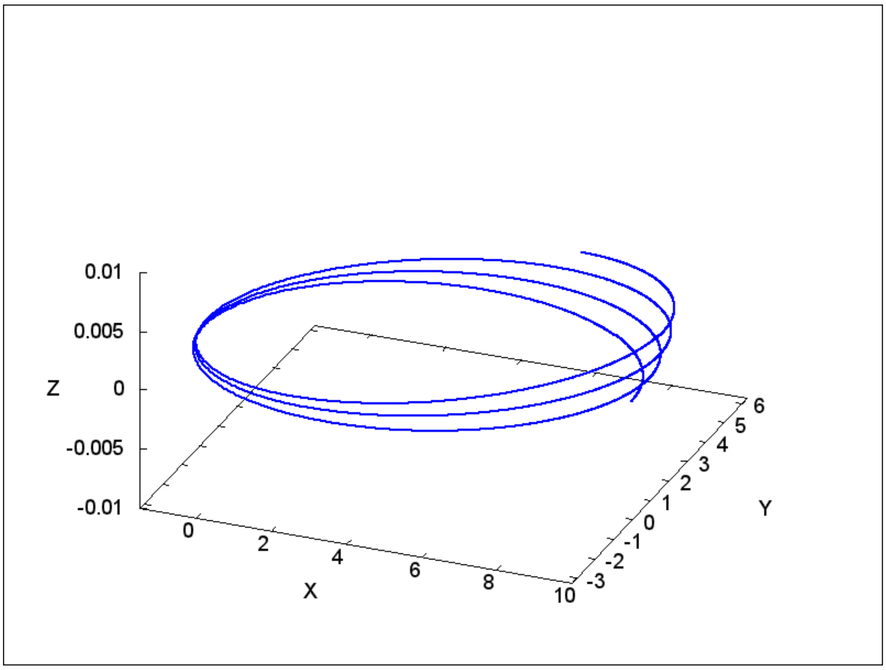


Figure 4: Precessing elliptic orbit due to fluid dynamics effects.

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ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension, and the staff of AIAS and others for many interesting discussions. The CEO of Annexa Inc., Dave Burleigh, is thanked for posting, site maintenance, and feedback software and maintenance. Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

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